APPLICATION OF BAYESIAN RELIABILITY CONCEPTS TO CRUISE MISSILE ELECTRONIC COMPONENTS

THESIS

Richard K. Lemaster, Captain, USAF

AFIT/GSM/LSG/89S-24

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GSM/LSG/89S-24

APPLICATION OF BAYESIAN RELIABILITY CONCEPTS TO CRUISE MISSILE ELECTRONIC COMPONENTS

THESIS

Richard K. Lemaster, Captain, USAF

AFIT/GSM/LSG/89S-24

The contents of the document are technically accurate, and no sensitive items, detrimental ideas, or deleterious information is contained therein. Furthermore, the views expressed in the document are those of the author and do not necessarily reflect the views of the School of Systems and Logistics, the Air University, the United States Air Force, or the Department of Defense.

AFIT/GSM/LSG/89S-24

APPLICATION OF BAYESIAN RELIABILITY CONCEPTS TO CRUISE MISSILE ELECTRONIC COMPONENTS

THESIS

Presented to the Faculty of the School of Systems
and Logistics of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Systems Management

Richard K. Lemaster, B.S.
Captain, USAF

September 1989

Approved for public release; distribution unlimited.

Acknowledgments

The research and writing of this thesis was a task that was a much more rewarding experience thanks to the efforts of several individuals. My faculty advisor, Lt Col Phillip Miller, knew when to leave well enough alone, and when to step in with a word of encouragement or clarification. Without his continuing patience and vision, this effort would still be in its prior stages.

I would like to thank Col Barnett and Lt Col Rigano in the head office of the Advanced Cruise Missile Program Office for their support in this endeavor. In particular, I call attention to the efforts of Mr. Dan Shaver, also in ASD/VC, in keeping the lines of communication open and helping me get access to data at the contractor's facilities. Mr. Dan Cunha at General Dynamics/Convair Division was especially helpful in the formative stages of the research.

Mr. Hugh Lynn graciously consented to provide me with the "expert information" that plays a critical role in that model and Dr. Stan Kaplan made time available to explain his theories. Finally, I'd like to express my deep appreciation for the sacrifices my wife Barb made to care for our new daughter Katie during those long nights that I was working on my posterior in my office.

Rick Lemaster

Table of Contents

List of Figures

List of Tables

AFIT/GSM/LSG/89S-24

Abstract

The purpose of this research was to evaluate the applicability of Bayesian statistical methods to the problem of determining cruise missile component reliability. There were three objectives: 1) to develop models incorporating Bayesian reliability concepts that can be used to predict component reliability based on data available in a program transitioning from development to production; 2) to determine the model's validity in comparison with classical statistical models; and 3) to assess the accuracy of both approaches against actual cruise missile flight test history.

A total of six models were developed for the failure rate of the Tomahawk Cruise Missile Guidance Set using both exponential and binomial distributions. The flight test data seemed to belong to another failure distribution, and was not useful as a measure of performance as had been proposed.

The Bayesian Expert Information Model provided reasonable point estimates of the failure rate and markedly shorter 90% confidence intervals. In general, the Bayesian models had confidence intervals that were shorter than the classical statistical inference models, allowing a more accurate decision-making process.

Future effort in this field could be directed toward applying these models to other weapon systems or components. Other applications could include using the Bayesian approach in the Aeronautical Systems Division Avionics Integrity Program (AVIP) to increase the Failure Free Operating Period of components, or in the assessment of the reliability of foreign technology based on partial information.

APPLICATION OF BAYESIAN RELIABILITY CONCEPTS TO CRUISE MISSILE ELECTRONIC COMPONENTS

I. Introduction

General Issue

The reliability of electronic components in United States Air Force weapon systems greatly contributes to the overall weapon system mission reliability. Before a decision to proceed with weapon system production can be made, the Air Force has an obligation to verify these reliability values or demonstrate that they can reasonably be expected to be attained during the early production testing of hardware. The type of statistical analysis used to establish the confidence in reliability predictions can influence the amount of testing necessary. The accepted Air Force approach to date uses "classical" statistical inference techniques that require large data samples to generate reliability predictions with confidence intervals constrained enough to be useful for management decisions.

In order to make a meaningful reliability prediction (one with a narrow band of high confidence) a large amount of test data must be accumulated. This usually occurs at a time when both funding and test assets are in critically short supply and the time required to accomplish the testing can threaten program production authorization. Testing is generally limited; unidentified component deficiencies, even if later detected, may impact operational readiness and mission reliability or necessitate costly retrofits.

Problem Statement

Can a reliability model based on Bayes' Theorem be developed that better predicts component reliability using minimal test data and provides the same or better levels of confidence than classical statistical inference methods?

Research Objectives

The objectives of this research are: 1) to develop models incorporating Bayesian reliability concepts that can be used to predict component reliability based on data available in a program transitioning from development to production; 2) to determine the model's validity in comparison with classical statistical models; and 3) to assess the accuracy of both approaches against actual cruise missile flight test history.

Scope

This research will be dealing with cruise missile electronic components. Cruise missiles are unique hybrids of missiles and aircraft in that they perform most of the functions common to manned aircraft without the characteristic designed redundancy that tends to increase aircraft mission reliability. Additionally, because they are largely dormant systems, there is relatively little cruise missile operational data for use in evaluating both system and component reliability. There are many different approaches that incorporate elements of Bayesian philosophy in statistical decision theory. The next section will identify and discuss some of these and contrast them with the more prevalent classical inference view.

II. Literature Review

Introduction

This literature review will consider current thinking in the use of Bayesian concepts for making reliability predictions of electronic components.

Justification

> The defense posture of the United States is based on countering a numerically superior enemy with the combat capability of qualitatively superior weapon systems. To win under these circumstances, systems must perform not just once but must sustain operational performance over time. (7:1)

This phrase defines reliability as it is used in this research and reflects a renewed Air Force commitment to ensuring reliability in weapon systems. Concurrent with the rising interest in reliability is an increase in spending on research, development, testing and evaluation of new weapon systems that consumed about one-sixth of the Department of Defense budget in 1983 (38:1). Any savings in the testing process that do not adversely impact reliability performance are clearly desirable.

Plan of Development

This review will first introduce the Classical Statistical Inference and Bayesian approaches from a managerial perspective. Reliability concepts and reliability testing requirements will be addressed, followed by some classical statistical concepts. It then will explain Bayesian statistical concepts and the various testing applications described in the literature, finally discussing the proposed model for this research.

Analysis of the Literature

Managerial Perspective. Air Force managers are tasked with the responsibility of making weapon system acquisition decisions. To provide themselves with a basis for a particular decision, they consult the accumulated expertise of engineers and statisticians. Often, there is a tendency on the part of the manager to rely too heavily on the analyses, making statistically significant decisions that are compromised by limitations of the statistical theory employed. Berger recounts an example from Pratt that demonstrates this:

> An engineer draws a random sample of electron tubes and measures the plate voltages under certain conditions with a very accurate voltmeter, accurate enough so that measurement error is negligible compared with the variability of the tubes. A statistician examines the measurements, which look normally distributed and vary from 75 to 99 volts with a mean of 87 and a standard deviation of 4. He makes the ordinary normal analysis, giving a confidence interval for the true mean. Later he visits the engineer's laboratory, and notices that the voltmeter used reads only as far as 100, so the population appears to be "censored". This necessitates a new analysis if the statistician is orthodox. However, the engineer says he has another meter, equally accurate and reading to 1000 volts, which he would have used if any voltage had been over 100. This is a relief to the orthodox statistician, because it means the population was effectively uncensored after all. But the next day the engineer telephones and says, "I just discovered my high-range voltmeter was not working the day I did the experiment you analyzed for me." The statistician ascertains that the engineer would not have held up the experiment until the meter

> was fixed, and informs him that a new analysis will be required. The engineer is astounded. He says, "But the experiment turned out just the same ... I learned exactly what I would have learned if the high-range meter had been available. (3:24-5)

The difference between the engineer and the statistician in the above example is one of philosophy. The statistician is subscribing to the classical statistical inference approach, while the engineer is intuitively describing the Likelihood Principle, which is the basis for the Bayesian approach. This principle will be explained in greater detail later in this review, but it in essence is stating that only actually observed samples have a bearing on the determination of a population characteristic. The key point lies in recognizing the utility of statistical theory as a tool for decision-making rather than as an end to the means.

Approach Contrasts. The fundamental difference between the two approaches can be seen in Figure 1 and Figure 2. Here, sampling theory refers to classical statistical inference theory.

```
   Assumptive
Sampling Model  <------------------------
      |                                 ^
      |                                 |
      v                                 |
 Sample Data ----> Inductive ----> Statistical
                   Reasoning        Inferences
```

Figure 1. Inferences Based on Sampling Theory (22:166-8)

```
Assumptive <-------------------------------------
 Sampling Model                                 ^
    |                                           |
    |                                           |
    v                                           |
 Sample --->|                                   |
  Data      |                                   |
            | ---> Bayes' --> Deductive --> Statistical
 Assumptive |      Theorem     Reasoning      Inference
  Prior     |
  Model --->|
```

Figure 2. Bayesian Inference (22:166-8)

These approaches will be described in more detail later, however, it can be seen that the Bayesian reasoning process is deductive as opposed to the classical inductive process. Martz also summarizes other characteristics of the two approaches in Table 1.

Another criticism of the classical approach is its use of what Berger calls "initial precision" as a measurement of accuracy. Before an experiment is run, the researcher somewhat arbitrarily selects a decision rule that defines the level of precision. A common decision rule has an initial precision of 90% when an α value = 0.10 is chosen. This is appropriate for a long series of identical tests, such as sampling products on a production line, where the unknown true characteristic being tested would fall within a given confidence interval in 90% of the whole series. In a one time test, the "final precision" is of greater interest,

Table 1. Summary of Certain Characteristics of the Sampling Theory and Bayesian Methods of Statistical Inference (22:169)

Characteristic	Sampling Theory	Bayesian
Parameter of Interest	Unknown Constants	Random Variables
Prior Distribution	Does not exist	Explicitly Assumed
Sampling Model	Assumed	Assumed
Posterior Distribution	Does not exist	Explicitly Derived
Reasoning	Inductive	Deductive
Type of Interval Estimate	Confidence Interval	Probability Interval
Past Experience	Not Applicable	Applicable
Purpose of Sampling Experiment	Supply data for Making Inferences	Confirm or Deny Expected Performance from Past Experience
Quality of Inferences	More Restrictive because of Exclusive Use of Sample Data	Depends on Ability to Quantitatively Relate Past Experience to Sample Data
Quantity of Sample Data	Bayes' approach usually requires less because it utilizes relevant past data	

i.e., what is the probability that the true value is contained in the 90% (or any other) confidence interval (3:18-9).

Berger identifies seven interrelated reasons for considering the Bayesian approach.

(i) Prior information is too important to ignore or deal with in an adhoc fashion.

(ii) According to most "classical" criteria, the class of "optimal" procedures corresponds to the class of Bayes procedures, so one should select from among this class according to prior information.

(iii) The Bayesian viewpoint works better than any other in revealing the common sense features of a situation and producing reasonable procedures.

(iv) The goal of statistics is to communicate evidence about uncertainties, and the correct language of uncertainty is probability. Only subjective probability provides a broad enough framework to encompass the types of uncertainties encountered, and Bayes theorem tells how to process information in the language of subjective probability.

(v) Axioms of rational behavior imply that any "coherent" mode of behavior corresponds to Bayesian behavior with respect to some prior distribution.

(vi) The Likelihood Principle seems irrefutable, yet the only general way of implementing it seems to be through Bayesian analysis.

(vii) Bayesian posterior measures of accuracy seem to be the only meaningful measures of accuracy. (37:2)

The mechanics of the classical statistical inference and Bayesian approaches will be addressed in greater detail. First, though, a general overview of reliability, probability distributions, and testing will provide background information for later discussions.

Reliability. Reliability is the probability that a system or component operates without failure through a specified time period (1:105). A failure is the partial or total loss of function of a unit in such a way that its intended purpose is impeded or stopped. An electric light bulb is an example of a unit with a well-defined failure mode (10:71). For the purposes of this research, a component's reliability will be evaluated in terms of its life and its ability to complete a specified task.

A component's life is defined as the difference in time between when a unit begins to function and the instant that a failure occurs (10:78). For a sample of components, this life may also be expressed as a failure rate, which is the ratio of failures occurring in a period of time to the number of units that had no failures up to that instant (1:105).

The failure rate has the general characteristics of a "bathtub curve" when plotted against time (see Figure 3). It starts with a short period of time with a large decreasing failure rate akin to sliding down the inside of a bathtub. This corresponds with a high failure rate due to "infant mortality" or production defects that cripple the component's function. The bottom of the bathtub curve refers to the useful life of a component; it has the smallest failure rate and failures during this period are frequently categorized as random because the failure rate is constant. At the end of the bathtub curve, the failure rate increases sharply corresponding to the increasing number of failures due to aging (20:84-6). Many component samples experience both the infant mortality and aging phases; techniques such as environmental stress screening have been

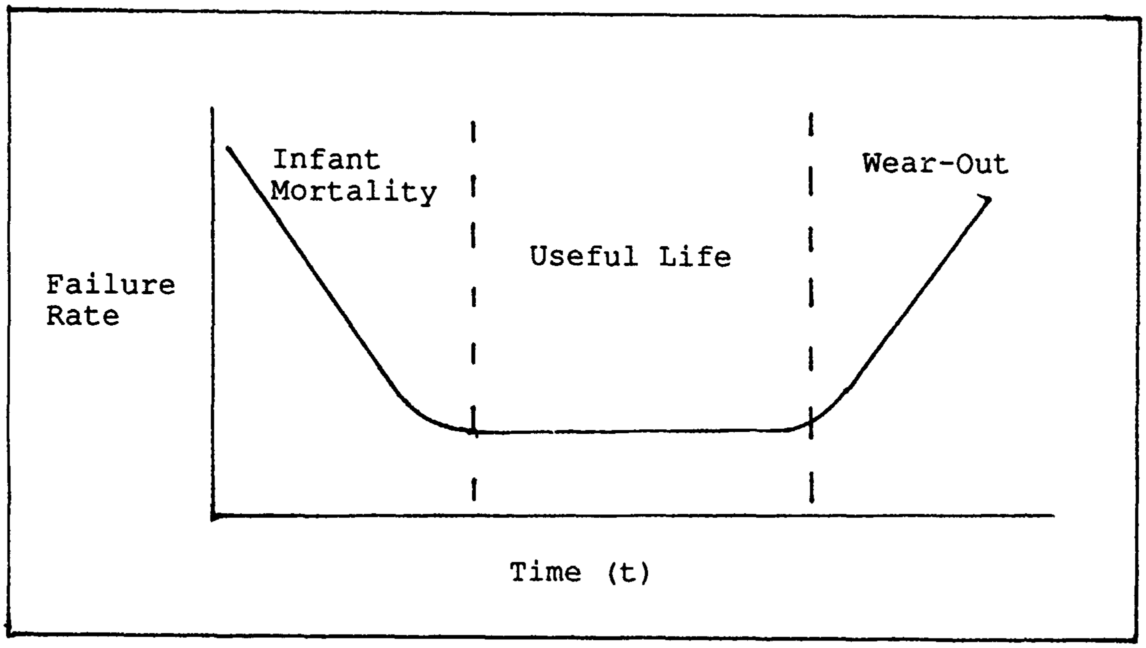


Figure 3. Bathtub Curve (20:84-86)

developed to handle the former and components can be removed from the inventory prior to the onset of the latter (15:3-5).

Probability Distributions. The failure rate for the useful life of a sample of components may be described as a random variable with one of several probability distributions: exponential, binomial, normal, gamma, and Weibull. The exponential distribution (1:106) has a constant failure rate, , that is the reciprocal of the Mean-Time-Between-Failure (MTBF), , and reliability,

$$R(t) = \exp(-t^{*}\ ) \quad (1)$$

The binomial distribution can be used when a dichotomy exists between the states a random variable can assume within a population. For reliability purposes, it can describe situations where components either successfully complete or fail an event. Two assumptions are required: 1) each of n sample items has the same chance p of being in a certain state, and 2) the outcomes of the n sample items are statistically independent (25:91-2).

For a sample of n units, the binomial probability function (25:92) can be expressed as

$$f(y) = \frac{n!}{y!\,(n-y)!}\, p^{y}\,(1-p)^{n-y} \qquad (2)$$

where y is the number of successes. In this case, p is the reliability of the component under test. The other distributions have specific applications that will not be discussed here (1:106-7).

"The instant at which a failure will occur cannot be told in advance: this instant is random (10:145)." Although we can describe the general failure probability distributions, reliability testing is necessary to estimate specific "numerical reliability characteristics (10:145)."

Reliability Testing. Testing is used to verify the validity of theoretical reliability calculations, usually in an environment as nearly representative of the operational scenario as is possible. Ling and Arsenault identify four underlying principles of reliability testing, including the use of "statistically efficient tests chosen to minimize cost and time to an accept/reject decision (1:336-7)."

MIL-STD-785 also states that reliability testing should be "tailored for effectiveness and efficiency (maximum return on cost and schedule investment) in terms of the management information they provide (8:A-25)." The military standard (8:A-25) points out a difference between reliability values measured in early Reliability Design Growth Testing (RDGT), which are not expected to correlate with operational values, and those from final RDGT and Reliability Qualification Testing (RQT) which must possess a clear traceability to field requirements.

Reliability Confidence Testing (RCT) is a shortened form of RQT sometimes used when funding and time do not permit a complete RQT as described by MIL-STD-781. The Cruise Missile Guidance Set Reliability Demonstration will be considered to be this kind of a test. The next section describes the hardware components and the objectives of this test.

Cruise Missile Guidance Set Reliability Demonstration. The Tomahawk Cruise Missile is an airbreathing turbofan missile designed to fly at high subsonic speed and low altitude, striking targets with high accuracy at distances up to 1500 nautical miles. There are conventional and nuclear variants developed for the U.S. Air Force and Navy designed to be launched from ground vehicles, ships, and submarines. This research will consider only the Tactical Land Attack Missile (TLAM) variants.

The heart of the TLAM missile is the Cruise Missile Guidance Set (CMGS), which consists of a gyro assembly, a radar altimeter, and an on board microprocessor/computer. Prior to launch, the launch location is loaded into the system and during flight the CMGS uses terrain correlation for navigation updating. While flying over land, the downlooking radar computes terrain altitude profiles that are compared with computer stored digital map data, providing the basis for corrections to the missile flightpath. The CMGS went through a substantial configuration change midway through production, partially due to accumulated producibility changes and minor reliability improvements. Based on a perceived reliability deficiency and lack of confidence in the configuration change, the Secretary of the Air Force directed that reliability demonstration testing be accomplished on both CMGS configurations prior to field retrofit (21).

The CMGS Reliability Demonstration had two objectives: 1) provide a point estimate of the CMGS mission MTBF, and 2) determine the field environmental effects on mission reliability. The test was divided into two phases corresponding to the objectives; the primary difference being the sequence and severity of the thermal and vibration cycles. These cycles were organized in such a fashion as to simulate the environment experienced by the CMGS in operational use. The sample to be tested consisted of units taken from the field and verified to be in working order prior to the test. Because of time and funding limitations, a limited number of sample units was used. In the event of a failure, the failed unit was repaired and put back into the test. Test data was collected according to both the number of mission scenarios successfully completed and total time in test for each test asset. Specific details are contained in the test plan (31:1.1-3.23).

Classical Statistical Inference

This section will provide some background material on the classical statistical inference approach.

> ...inferential statistical analysis is concerned with measuring the characteristics of only a sample from the population and then making inferences, or estimates, about the corresponding value of characteristics in the population from which the sample was drawn. (13:9-10)

The total number of components in use in all of the weapon systems can be defined to be the population. From this population, a subset, or sample, of components can be selected for testing when it is not practical to test or measure each population member. This is certainly the case when trying to determine the reliability of one-time use items like cruise missiles. The sample is selected at random, i.e. each member of the population has an equal chance of being included in the sample, to validate inferences that may be made to the population parameters.

There are two types of inferences that may be drawn from sample statistics to population parameters -- point estimates and interval estimation. A point estimate is a single value that best approximates the true value of the population parameter, but does not indicate how much uncertainty is present. An interval estimate can specify a probability that the sample statistic is within a certain interval about the true population parameter. A point estimate, such as the mean of a distribution, $\bar{x}$, will differ from the population parameter, μ, by some unknown amount of sampling error, $\bar{x} = \mu \pm \varepsilon$. If the sample size is large, the sampling distribution and the distribution of sampling errors will both be nearly normal by the Central Limit Theorem. This results in an expression for the confidence interval based on the population standard error,

$$(\bar{x} - z^*\sigma) < \mu < (\bar{x} + z^*\sigma) \qquad (3)$$

where z is the critical value associated with a specified probability. In most cases, the population standard error is not known, and must be estimated using the sample standard error, s. The interval estimate obtained using s is called an approximate confidence interval (13:135-143).

Bayesian Statistics

This section will discuss Bayes' Theorem and several approaches toward its implementation in reliability testing. One model, in particular, will be described that seems to be useful for this research.

Bayes' Theorem. The Bayesian technique is easy to explain, but its ramifications are more complex.

> Suppose that there exists a set of mutually exclusive and exhaustive events that are considered: it is known in advance that one, and only one, of these events will actually occur, but there is uncertainty about which of these it will be. One begins by assigning a probability to each of these events on the basis of whatever evidence is currently available. Then if additional evidence is subsequently obtained, the initial probabilities are then revised on the basis of this evidence by means of Bayes' Theorem. The initial probabilities are known as prior probabilities in that they are assigned before the acquisition of the additional evidence bearing on the problem. The evidence on which these probabilities are based is therefore prior information.... The probabilities which result from the revision process are known as posterior probabilities. (24:1-2)

Bayes theorem (4:10) can be expressed by the following equations:

$$P(\lambda|y) = \frac{P(y|\lambda)P(\lambda)}{P(y)} \qquad (4)$$

where $P(\lambda|y)$ is the posterior distribution of the population given sample data y,

$P(\lambda)$ is the prior distribution of λ,

$P(y|\lambda)$ is the likelihood function of λ given sample data y, and

$P(y)$ is a normalizing constant necessary to make the posterior distribution integrate or sum to one.

$$P(y) = \begin{cases} \int P(y|\lambda)P(\lambda)d\lambda & \lambda \text{ continuous} \\ \sum P(y|\lambda)P(\lambda) & \lambda \text{ discrete} \end{cases}$$

<u>Likelihood Principle</u>. In equation (4) above, the posterior distribution is proportional to the prior distribution multiplied by the likelihood function. Here the likelihood function is defined as the probability of observing the sample data y given the true population λ.

> The likelihood function...plays a very important role in Bayes' formula. It is <u>the</u> function through which the data y modifies prior knowledge of λ; it can therefore be regarded as representing the information about λ coming from the data. (4:11)

<u>Prior Distributions</u>. The continuous prior distribution is often discretized to allow more tractable data manipulation. Chay contends there is no longer any need to approximate continuous prior distributions through discretization because current computer calculations have made their numerical integration accessible. Additionally, care must be taken in subdividing the distribution into discrete masses to ensure the areas of large mass accurately portray true values (5:218). Conjugate distributions are often used for priors because the posterior distributions can be obtained relatively simply using analytical tools. Again alluding to the availability of computational tools,

Chay makes the point that alternate distributions should be considered as priors (5:216).

Martz and Waller identify characteristics of successful Bayesian reliability analyses that include:

1. A detailed analysis of the prior that will be used with documentation of data sources;

2. Use of a preposterior analysis to test the prior with hypothetical data;

3. Clear, definitive descriptions of the posterior distribution, with a sensitivity analysis of the resulting Bayesian inferences with respect to the original prior (22:189).

The preposterior analysis is used to assess the impact that conflicting, contradictory, or confirming data samples have when applied to the proposed prior. It consists of the following steps:

1. Using the proposed prior, consider two sets of hypothetical test data that seem to be likely and unlikely.

2. Compute the resulting posterior distributions using Bayes' theorem for each case.

3. Examine the posterior distributions for reasonableness based on the hypothetical test data. If the results seem inconsistent, adjust the prior and repeat the preposterior analysis (22:187). This adjustment often is the target of allegations of test data manipulation. It is important to keep the perspective that Bayesian reliability

analyses, like those used in classical statistics, are tools for decision making, rather than expressions of an absolute truth.

Bayesian Reliability Testing Approaches. This section will address the current approaches in implementing Bayesian concepts in reliability testing.

Classical testing was contrasted with the Bayesian approach by Wonnacott. He pointed out the classical dependence on the specified α or Type I error associated with rejecting a true hypothesis. Often, the level of acceptable error is less easily determined than the hypothesis value being tested (37:149-157). McCrory made an impassioned plea for the use of Bayesian techniques to save "reliability testing from an early death by providing a means to measure and express reliability quantitatively in a more timely and cost effective fashion (23:20)."

Launer and Singpurwalla used Bayes' Theorem to propose a model for detecting a deterioration in the system reliability of one-shot missiles. Additionally, they wanted to determine the marginal change in reliability since the last test and to minimize the number of samples used for the destructive testing (19:23-6). Another researcher working with one-shot devices proposed a series of reliability test plans which "...will be Bayesian in the sense that relevant data from previous testing is to be used in attempt to reduce the sample size required in subsequent testing

(38:3)." He defined producer's and consumer's risks and developed confidence intervals for the reliability predictions (38:8-14).

Three research studies have addressed the problem of determining system reliabilities based on component or subsystem data. Lampkin has considered the use of several expansion techniques to maintain precision in the calculation of exact Bayesian intervals as the number of subsystems in the reliability estimate increases (18:313-7).

Winterbottom was also interested in estimating Bayesian intervals for system reliabilities based on component data. He used exact and approximate methods both in a series of structures (series, parallel, quorum, and others) and in the case of mixed testing of components, where some are tested pass/fail and some have exponentially distributed failure times (36:1-43). Barlow has identified a couple of fast algorithms to calculate Bayesian system reliabilities based on individual component reliabilities. He investigated the effect of increasingly large numbers of components within networks (2:375-383).

A Rome Air Development Center report discusses the Bayesian approach to structuring reliability tests. It defines the producer's and consumer's risks and provides guidelines for selecting test risks. Tne report also compares classical statistical tests with Bayesian tests and concludes that the condition of the prior probability data will dictate which approach is optimum (6:5-35).

Johnson and Utts were concerned with estimated Bayesian means and confidence intervals of a normal population using contaminated data. They used improper priors, which are also relatively non-informative, and derived a posterior that is approximated by a student distribution with specific degrees of freedom, location and dispersion. The means and confidence intervals were demonstrated to be very robust and tolerant of outliers (12:8-16).

Kaplan proposed a "two stage" Bayesian procedure that could be used to develop a prior when there is little generalized industry data available. In his experiment, there was a machine failure rate that could be determined from three types of information. The first type, engineering knowledge of the design and construction of the machine, was used to prepare the first stage prior. This prior was updated by the second type of information, past test experience of similar machines, to form the first stage posterior. This posterior was then used as the second stage prior and updated to form a posterior with the third type of information, current machine test data. The first prior was the only distribution based on subjective rather than quantitative inputs (16:1-3).

Kaplan also explored the use of Bayesian probabilistic analysis to determine the failure rate of components in nuclear power plants. There, he also assumed a probability distribution was provided by an expert source for use as the prior. Uniform and nonuniform populations were considered along with constant and age dependent failure rates (14:5-20).

Expert Information Model. A model has been proposed by Kaplan for determining the reliability of electronic "black boxes". In its original form, it is a variation of a binomial approach. This research will also consider a derivative using an exponential distribution. A summary of the binomial approach will be addressed first.

> Our real interest ultimately, of course, is the reliability of the complete system. This will become known finally only from experience (i.e. tests and operations), with the complete system itself. However, such tests and operational data are usually very expensive. Moreover, we often need to predict the reliability of the complete system at early stages of design and production, before any (or certainly many) full system tests are done....It is often the case that, while we have little experience with the complete system, we have considerably more information about the individual boxes composing the system. (15:1)

The model is first introduced as a thought experiment in which a large number of identical boxes, N_0, are repeatedly put through an operationally realistic duty cycle, or "mission". The number of boxes surviving a given cycle i, N_i, divided by N_0 can be plotted against i cycles on semilog paper to obtain a graph that represents the 3 phases of a component's life -- infant mortality, useful life, and wearout (see Figure 4).

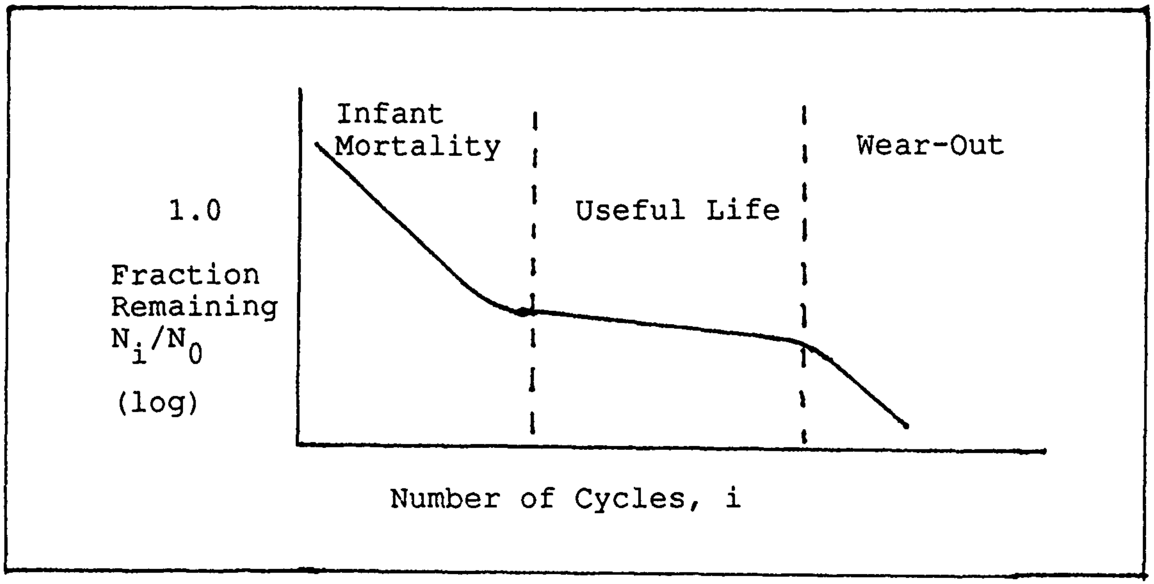


Figure 4. Survival Curve for Boxes (15:3)

This model assumes the useful life portion of the graph has a slope, λ , that is "the basic 'failure rate' of the box measured in units of failures per box per cycle (15:3)."

$$= \frac{N_i - N_{i+1}}{N_i} \quad \text{(i and i+1 in useful life)} \qquad (5)$$

Note that this definition is not the same as was used earlier in Figure 3 for the exponential model, where λ was measured as a function of operating time. It also assumes that the slope, and therefore λ, is nearly constant over the useful life of the box.

Kaplan (15:7) establishes a discretization of the distribution of lambda values from 0 to 1.0 for use in the following Bayesian expression:

$$p_j(\lambda|E) = \frac{p_o(\lambda_j)p(E|\lambda_j)}{{}_j\ p_o(\lambda_j)p(E|\lambda_j)} \tag{6}$$

where,

$p(\lambda_j|E)$ = the posterior probability that the true value of λ is λ_j, conditional on evidence E,

$p_o(\ _j)$ = the prior probability that the true value λ_o is λ_j before evidence E existed,

E = the evidence or information discovered, and

$p(E|\lambda_j)$ = the likelihood that given the true value of λ is λ_j, the evidence E would be obtained.

The first model uses an exponential distribution as the likelihood function. Assuming N_t is the number of boxes that remain unfailed at time t, and N_0 is the number present at time zero, then $N_t = N_0\, e^{-\lambda t}$ and there is a function f(t) such that $f(t) = Nt/N0 = e^{-\lambda t}$. This is the fraction of systems that remain unfailed at time t (17:11). The probability of k failures in time t can be represented by the Poisson distribution (27:8).

$$p_k(t) = \frac{(\lambda t)^k\, e^{-\lambda t}}{k!} \tag{7}$$

This yields the following Bayesian posterior expression for the exponential model (17:25):

$$p(\lambda|E) = \frac{p(\lambda_j)(\lambda_j t)^k e^{-(\lambda_j t)}}{\sum_{i=1}^{n} p(\lambda_j)(\lambda_j t)^k e^{-(\lambda_j t)}} \tag{8}$$

The likelihood function (15:7) for the binomial model can be expressed for E = {k failures in N cycles} as

$$P(E|\lambda_j) = \frac{N!\ (\lambda_j)^k\ (1-\lambda_j)^{N-k}}{k!(N-k)!} \tag{9}$$

Kaplan identifies some possible types of evidence that may provide information about the reliability of boxes (see Table 2). The following sections describe Kaplan's approach to using each of these evidence sources. In each case, evidence that is considered less precise is used to form a prior distribution that is then updated based on the new evidence.

Type E1 - this is data that is presumed to be most representative of the true box failure rate. The prior probability distribution would be based on evidence types E2...E11 and updated using the failure rate calculated from E1 data (either binomial or exponential) to estimate λ (15:8).

Type E2 - this is data that was obtained from simulated testing. A similar process is followed in using a prior distribution generated from evidence E3...E11, but in this case, the posterior distribution will be more representative

Table 2. Possible Types of Evidence
Relevant to Box Reliability (15:2)

Evidence	Description
E1	Direct experience of performance of the box, as part of the complete system in actual use.
E2	Experience of performance of the box, as part of the complete system, but not in actual use.
E3	Experience with performance of the box in isolation, outside the system, simulating actual use.
E4	Performance of the box in screening tests.
E5	Performance of the box in other tests.
E6	Knowledge of the reliability of subcomponents.
E7	Performance of earlier versions or designs.
E8	Experience of the same or similar boxes in other operating environments.
E9	Calculations, physics of failures, analyses.
E10	Expert judgements, general engineering knowledge, failure mechanisms, etc.
E11	Other.

of a failure rate λ_2 for the box in a simulated environment. Accordingly, the resulting posterior distribution may be modified by engineering judgement to reflect the increased uncertainty before it is used as a prior for evidence E1 (15:13).

Type E3 - this data generally comes from testing such as Production Reliability Acceptance Testing (PRAT) and RQT/RCT. PRAT is used to verify that production processes nave not degraded the designed reliability that was verified through RQT/RCT. The former consists of a large number of boxes tested once, while the latter is usually a small number of boxes tested repeatedly. In both cases, the focus is on the number of cycles successfully completed. As for evidence E2, the additional uncertainty due to differences in testing can be abrogated through modification of the posterior before it is used as a prior (15:14-16).

Type E4 - this data usually reflects the effects of an environment that is more severe than required operationally. As described earlier, manufacturers use Environmental Stress Screening to precipitate failures that will occur in the infant mortality period. The process is repeated until a unit passes through the cycles without a failure. ESS can then be defined as consisting of a burn-in cycle (which should have a significantly greater λ) and a failure free cycle (which should have a λ closer, but still higher than the operational λo value). Rather than using these to update a prior distribution from other evidence, it is more useful to plot the distributions as upper bounds for (15:17-20).

Type E5 - this type of data can be handled various ways, depending on the characteristics of the testing. If it is a box level acceptance test at ambient conditions, it may be possible to establish the lower bounds of λ for use in the same fashion as evidence E4. If the test is more representative of the operational environment, it can be used to update a prior based on evidence E6...E11 and form the prior for evidence E3 (15:21).

Type E6...E11 - this data is usually less precise than earlier test data. However, the use of every available piece of information is totally in consonance with the Bayesian philosophy. The specific application of each type of analysis or prior experience with an earlier generation of the box will depend on the circumstances. Suffice to say, the resulting distribution should be fairly broad to reflect its highly uncertain nature (15:22-23).

Having defined the problem area and some of the ongoing work in the field of Bayesian statistics, it is logical at this time to proceed to a discussion of the methodology followed in this research.

III. Methodology

Overview

This chapter describes the methodology used to accomplish the above stated research objectives. This brief overview of the methodology will be followed by descriptions of the data which will be analyzed. The classical statistical inference approach will be examined from two points of view: as a binomial distribution model, and as a constant failure rate exponential distribution model. In addition to the binomial and exponential distribution models referenced above, the Bayesian reliability analysis will also consider an expert information model. A discussion of the types of data available as sources to a Bayesian expert information model will be followed by a description of the steps involved in developing the model itself. A comparison of the point estimates and confidence intervals obtained from all the Classical and Bayesian models will be made with respect to those obtained from actual flight test history.

Data Sources

The data obtained from the CMGS Reliability Demonstration will be used to make statistical inferences about the true failure rate and reliability of the CMGS. The three phases of this test will each be examined separately and finally combined into a single data set in each of the following six different models: 1) the classical exponential, 2) the classical binomial 3) the Bayesian exponential, 4) the Bayesian binomial, 5) the Bayesian expert information exponential, and 6) the Bayesian expert information binomial models.

Flight Test Data

The flight test database consists of data from three variants of the Tomahawk Cruise Missile which use essentially common configurations of the CMGS. Flight test data for the current configuration will be used to calculate a point estimate and confidence interval for the true failure rate and reliability. Because the data can be evaluated both in terms of flight time before failures and number of successful flights, both exponential and binomial distributions will be used.

The flight times have been accurately recorded, but those times are not true indicators of the CMGS unit ages. The CMGS computer does have an Elapsed Time Indicator (ETI), but the clock is not reset every time the unit is refurbished and there appears to be little correlation between the failure times and the ETI clock readings. Since the exponential model is based on a constant failure rate, the true age of the unit is not required as long as it can be determined to be in an operational condition before the flight test occurs. This would be sufficient to establish the baseline from which estimation of the failure rate could proceed.

The flight test data will be evaluated to attribute each failure to a specific component. Components will be presumed to have been functional in all successful flight tests or flight test failures due to other components or factors. Assuming the missile achieved its system level reliability requirement, the flight test results will be presumed to be an accurate reflection of the component's operating environment and mission reliability.

Flight Test Exponential Point Estimate. In order to use an exponential distribution, an evaluation of the flight test failure data must be made. Only the failed CMGS units and their associated failure times will be compared with a hypothetical random distribution derived from the sample failure rate, R/T, as in (12). The sample failure rate includes test time from units under test that did not fail. A Kolmogorov-Smirnov Goodness of Fit test will be run to determine whether the failed items follow an exponential distribution. If there is not sufficient cause to reject the assumption that it is an exponential distribution, then the sample failure rate will be used as the maximum likelihood estimator for λ.

Flight Test Exponential Confidence Interval. The same confidence interval equation will be used to estimate the interval for 90% confidence as in equation (13) above. The confidence limits for the exponential distribution (22:122) may be approximated using the $X^2(n)$ distribution where X^2 (n) is the 100(1-α) percentile of the $X^2(n)$ distribution. The 100(1-α)% TCI for is

$$\frac{X^2_{\alpha/2}(2R)}{2t} ; \frac{X^2_{1-\alpha/2}(2R+2)}{2t} \tag{23}$$

Models

The first two classical models, (1 & 2), will use only the CMGS Reliability Demonstration data to derive point estimates and confidence intervals for the true failure rate of the population. The Bayesian exponential and binomial models, (3 & 4), will use early flight test results as the basis for the priors. They will then be updated with the CMGS Reliability Demonstration data to form the posterior distributions, from which point estimates and confidence intervals will be obtained. The Bayesian expert information models, (5 & 6), will consider a variety of data sources as described by the Kaplan model, (see Table 2), to establish a series of priors culminating in a final prior. The initial prior will be established based on information evaluated by an expert source. As in models (3 & 4), the prior distribution will be updated with the results of the CMGS Reliability Demonstration, using both exponential and binomial distributions. Point estimates and confidence intervals for the failure rate will be determined form the resulting posterior distributions. The mechanics of calculating the classical and Bayesian approaches will be as described in the following sections.

Classical Models

A reliability prediction will be made based on a point estimate of the sample data, using a maximum likelihood estimator. The uncertainty pertaining to the prediction will be described in terms of a two-sided confidence interval. The predictions will be accomplished using two assumed probability distributions. The first will be an exponential distribution with a constant failure rate and the second will be a binomial distribution. The total time in the test and the corresponding failures will be used for the exponential distribution, while the successful completion of each individual portion of the tests will be scored as events for the binomial distribution. The sample data will be fit to the distributions using a Komolgorov-Smirnov (KS) Goodness of Fit test.

The Classical Exponential Approach. Assuming the data are distributed exponentially, point estimates for the failure rate, λ, and a 90% confidence interval about λ will be calculated.

Classical Exponential Point Estimate. The maximum likelihood estimator for the exponential distribution is R/T where R is the number of failures, presumed to be a random variable, and T is the total time on test.

<u>Classical Exponential Confidence Interval</u>. The confidence limits for the exponential distribution may be approximated using the $X^2(n)$ distribution where $X^2_\alpha(n)$ is the 100(1-α) percentile of the $X^2(n)$ distribution.

> In the case of Type I/time truncated testing with replacement, the sufficient statistic R (the random number of failures) follows a Poisson distribution... and the relationship between the Poisson and the $X^2(n)$ distributions permits approximate inferences to be based on the $X^2(n)$ distribution. (22:120-1)

The 100(1-α)% TCI for (22:122) is

$$\frac{X^2_{\alpha/2}(2R)}{2t} ; \frac{X^2_{1-\alpha/2}(2R+2)}{2t} \qquad (10)$$

<u>The Binomial Approach</u>. The binomial distribution has a maximum likelihood estimator p' such that the probability of survival is the ratio of X survivors from a set of n units placed into a test of given duration (22:53).

$$p' = \frac{X}{n} \qquad (11)$$

The 100(1-α)% TCI may be approximated using a transformation to the F distribution (22:56) given by the formula

$$\frac{x}{x+(n-x+1)F_{1-\alpha/2}(2n-2x+2,2x)} ;$$

$$\frac{(x+1)F_{1-\alpha/2}(2x+2,2n-2x)}{(n-x)+(x+1)F_{1-\alpha/2}(2x+2,2n-2x)} \quad (12)$$

Bayesian Models

The Data Sources. As a weapon system is transitioning from development to production, there is both a wealth and paucity of available data upon which to base reliability predictions. Generally, the engineering department is long on analyses and comparisons to existing systems, while the test department has little in the way of operational data; being limited chiefly to testing with engineering prototypes. There is little disagreement that neither of these by themselves constitute a data base with high validity. It is possible to identify other data sources which when compiled in a model can produce a more comprehensive picture of component reliability. These include:

1) operational system level test data,
2) system level acceptance testing,
3) component level acceptance testing,
4) environmental stress screening data,
5) qualification testing data,
6) design analyses, such as Mil-Hdbk-217D,
7) comparisons to preceding weapon systems.

The conservative application of the Bayesian approach uses old flight test data to develop the prior. In the event of insufficient flight test data, system level and possibly box level acceptance test data could be used. Acceptance testing may not be representative of the operational environment, and the objective use of these types of test data can introduce additional risk to the analysis.

The Bayesian Exponential Approach. Assuming an exponentially distributed failure rate, the distribution of failures in a fixed total test time can be described by the Poisson distribution (22:255).

$$p(\text{s failures in total time } t \mid \lambda) = \frac{e^{-\lambda t} * (\lambda * t)^{s}}{s!} \qquad (13)$$

where, $\lambda, t > 0$, $s = 0, 1, 2, \ldots$. Because not all of the flight test units failed during the test and failed units were replaced by new units for subsequent tests, the flight test data may be considered to be Type I, time truncated, with replacement (22:119).

Bayesian Exponential Prior. This Poisson sampling process can use a gamma prior distribution to achieve maximum flexibility and ease the mathematical burden of calculating the posterior distribution. Because of these two reasons, the gamma(α,β) prior distribution (22:289) is one of the most widely used for λ, and has the following probability density function:

$$g(\lambda;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} * \lambda^{\alpha-1} * e^{-\beta\lambda} \qquad (14)$$

where λ, α, and $\beta > 0$.

The shape parameter, α, and the scale parameter, β, may be interpreted as the "pseudo number of failures" and "pseudo length of time" for the prior life test. The "pseudo" refers to the supposition that these are the values that would best fit a prior appropriate for the experimental data (22:289).

The simplest estimates for α and β are the number of flight test failures and total test time.

Bayesian Exponential Preposterior Analysis. A preposterior analysis will be conducted using two hypothetical samples of data. The first set, considered likely, is one failure in 256 hours, the CMGS MTBF (29:1D). The unlikely data set consists of ten failures in 256 hours. Posteriors for each will be calculated using the gamma (α,β) prior distribution and evaluated for realism. The prior distribution will be adjusted if necessary.

Bayesian Exponential Posterior. The resulting posterior distribution for the gamma(α,β) prior distribution is gamma$(s+\alpha, t+\beta)$, where s and t are the number of failures and total test time from the CMGS Reliability Demonstration. The probability density function (22:290) is described by

$$g(\lambda | s;\alpha,\beta) = \frac{(t+\beta)^{s+\alpha}}{\Gamma(s+\alpha)} * \lambda^{s+\alpha-1} * e^{-(t+\beta)\lambda} \tag{15}$$

where $\lambda > 0$.

Bayesian Exponential Point Estimate. The Bayesian point estimator is the mean of the posterior distribution, which, under the squared error loss function, is

$$E(\lambda | s;\alpha,\beta) = \frac{s + \alpha}{t + \beta} \tag{16}$$

where $s + \alpha$ is the combined number of failures and $t + \beta$ is the combined total test time (22:292).

<u>Bayesian Exponential Probability Interval</u>. The Total Bayesian Probability Interval (TBPI) can be constructed by using chi square (X^2) percentile values for $(2s + 2\alpha)$. A 90% confidence interval will be calculated for comparison with intervals from the other models. The upper and lower interval endpoints for the $100(1-\alpha)$% TBPI (22:294) are

$$\frac{X^2_{\alpha/2}(2s + 2\alpha)}{2(t + \beta)} \; ; \; \frac{X^2_{1-\alpha/2}(2s + 2\alpha)}{2(t + \beta)} \tag{17}$$

<u>The Bayesian Binomial Approach</u>. For the binomial case, the family of $\beta(x,n)$ distributions will be used to calculate both the prior and posterior distributions because they are relatively easy to calculate and may be adapted to a number of situations. For example, the selection of x and n can be based on the analyst's knowledge and experience with the system. This can have the effect of increasing or decreasing the importance of the prior on the resulting posterior distribution.

Bayesian Binomial Prior. The $\beta(x_0,n_0)$ distribution is a conjugate prior for the binomial sampling model. The choice of x_0, the "pseudo number of survivors," and n_0, the "pseudo sample size," can be made on the basis of the analyst's prior knowledge, or based on the results of earlier objective testing.

> Weiler (1965) shows that the effect of assuming a $\beta(n_0,x_0)$ distribution, when in fact the true prior distribution is not of the beta type, is negligible in many practical applications. He shows that rather severe deviations in the beta prior parameter values produce only slight changes in the corresponding posterior distributions. (22:265)

For this research, earlier flight test data will be used to estimate values for x_0 and n_0 that fit the $\beta(x_0,n_0)$ distribution. The simplest estimate for x_0 is the number of CMGS units that have successfully flown in flight testing. Because there was a significant configuration change in the CMGS computer, only flight test data pertaining to the newer configuration will be used to develop this prior distribution. Similarly, the simplest estimate for n_0 is the number of guidance sets that have been flight tested.

Bayesian Binomial Preposterior Analysis. As above for the Bayesian exponential approach, a preposterior analysis will be conducted using two hypothetical data sets. The likely sample will consist of 99 successes out of 100 attempts and the unlikely sample will be 90 successes out of 100 attempts. An evaluation of the prior will be made based on these results.

Bayesian Binomial Posterior. The resulting posterior distribution is also a beta distribution of the form $(x+x_0, n+n_0)$, where x and n are the observed survivors and sample size from the experiment being used to update the prior. In this case, x and n will be the number of missions successfully completed and the number attempted, respectively. The probability density function (22:266) of the new posterior distribution is

$$g(p|x;x_0,n_0) = \frac{\Gamma(n + n_0)}{\Gamma(x + x_0)\ \Gamma(n + n_0 - x - x_0)} * p^{(x+x_0)-1} * (1-p)^{(n+n_0-x-x_0)-1} \qquad (18)$$

where $0 < p < 1$.

Bayesian Binomial Point Estimate. A point estimator for the probability, p, which represents reliability in this sampling scenario, can be determined from the posterior mean (22:267).

$$E(P|x;x_0,n_0) = \frac{x + x_0}{n + n_0} \qquad (19)$$

Bayesian Binomial Probability Interval. Again assuming a $\beta(x_0,n_0)$ prior, the Total Bayesian Probability Interval (TBPI) can be constructed from a transformation to the $F(n_1,n_2)$ distribution. A 90% confidence interval will be calculated for comparison with intervals from the other models. The upper and lower interval endpoints (22:270) for the $100(1-\alpha)\%$ TBPI are

$$\frac{x + x_0}{x+x_0+(n+n_0-x-x_0)F_{1-\alpha/2}(2n+2n_0-2x-2x_0,\ 2x+2x_0)};$$

$$\frac{(x+x_0)F_{1-\alpha/2}(2x+2x_0,\ 2n+2n_0-2x-2x_0)}{n+n_0-x-x_0+(x+x_0)F_{1-\alpha/2}(2x+2x_0,\ 2n+2n_0-2x-2x_0)} \quad (20)$$

<u>The Expert Information Approach</u>. The Kaplan model described earlier will be used with some of the above data sources to make the most accurate reliability prediction possible. The chief advantage in using Bayes' Theorem in a predictive model is its ability to incorporate data normally excluded as being "subjective" in classical approaches. This data is analyzed and the appropriate subjective probability distribution (SPD) describing the data is used to develop the prior distribution. The SPD is a discrete distribution whose total area sums to one. Expert information can be used to evaluate each of the data sources used in preparing and combining the prior distributions to form the final prior distribution.

Both the exponential and binomial approaches will be investigated using the following Bayesian relationships:

$$p(\lambda_i|X) = p(\lambda_i)\ \frac{p(X|\lambda_i)}{p(X)} \quad (21)$$

where,

$p(\lambda_i|X)$ = probability of failure rate λ_i, given the experience X

$p(\lambda_i)$ = probability of λ_i, prior to having X

$p(X)$ = prior probability of X, also

$$= \sum_{\lambda_i} p(X)p(X|\lambda_i), \text{ for all discrete values of } \lambda_i$$

for the exponential:

$p(X|\lambda_i)$ = probability of X, for each given λ_i,

$$= \frac{(\lambda_i * T)^n}{n!} * \exp(-\lambda_i * T),$$

with n failures in time T,

for the binomial:

$p(X|\lambda i)$ = probability of X, for each given λi,

$$= \frac{n!}{k! * (n-k)!} * (\lambda_i)^k (1-\lambda_i)^{n-k}$$

with k failures in n attempts.

Bayesian Expert Information Prior. Using the definitions of evidence presented by Kaplan in Table 2, the following data will be used to develop the iterative priors.

Step 1. Type E8 - The computer used in the CMGS is its most complex component and bears the greatest allocated reliability requirement. It is also very similar to the Inertial Navigation Element used in the Air Launched Cruise Missile for the past six years. A subjective evaluation of the CMGS reliability will be based on a comparison of ALCM INE field performance data (35) to its reliability specification. For example, if the ALCM INE has a demonstrated MTBF of 500 hours versus a specification value of 400, then the probability distribution for the CMGS could be adjusted upward from its specification value of 300 to a value 25% higher.

The specific amount of adjustment will be based on an estimate provided by Mr. I. Hugh Lynn, the former GLCM Chief Avionics Engineer. He was responsible for conducting technical evaluations during the CMGS Reliability Demonstration. Mr. Lynn has been a practicing government engineer for the past 22 years and is currently the Computer Resources Branch Chief (ASD/ENASC) for the Aeronautical Systems Division, Wright Patterson AFB, OH. His experience is typical of Department of Defense individuals who would be tasked to make this type of an evaluation for current weapon systems (21).

Issues to be considered include how many components in the CMGS do not have comparable functions in the INE and how the ALCM and Tomahawk cruise missile environments compare. The MTBF estimates will be converted to failure rates to make the comparisons more tractable. Because there is uncertainty in these estimates, a range of failure rates with associated probabilities can be plotted to form the SPD. This distribution is the First Prior (see Figure 5).

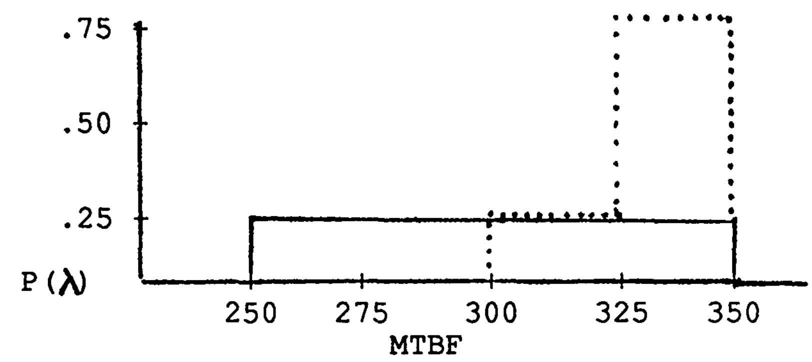


Figure 5. Subjective Probability Distribution (SPD) for the CMGS Based on ALCM INE Field Data

Step 2. Type E6 - CMGS design weaknesses identified in the Ground Launched Cruise Missile Flight Critical Item Investigation (11) are the basis for updating the First Prior distribution. The informed source (Mr. Lynn) evaluates the impact of each weakness on the designed reliability to develop a likelihood that can be used to update the First Prior. The resulting posterior distribution is used as the Second Prior for the next iteration involving simulated testing.

Step 3. Type E2 and E4 - Performance of the CMGS in acceptance testing and failure free Environmental Stress Screening (ESS) testing is used to update the Second Prior to reflect the performance of the CMGS in simulated environments. Additionally, CMGS performance in early ESS cycles will be evaluated as a lower bound on MTBF. This is due to the presence of manufacturing defects that are precipitated as failures in early ESS temperature and vibration cycles. The design posterior distribution is updated by this simulated testing SPD to create the Third Prior.

At this point, the analysis will independently pursue both exponential and binomial approaches to continue developing the final priors and estimate CMGS failure rates. The SPD used for the Third Prior, as well the resulting posterior distributions will be updated using Bayes' Theorem and the Poisson model for the exponential approach. For the binomial approach, Bayes' Theorem and equation (14) will be used.

Step 4a. Type E1 - (Exponential) - The Third Prior distribution is updated by flight test results to form the Final Prior. The flight test database will be examined to ensure only flight data from comparable CMGS configurations is included in the analysis. These configurations are defined as those guidance sets manufactured in the fiscal year FY 1983 and since then. The number of failures and total mission time will be used to obtain probabilities for each failure rate, λ_i. These probabilities form the Final Prior.

Step 4b. Type E1 - (Binomial) - The Third Prior is similarly updated by flight test results in the form of the numbers of successes and attempts. Equation (14) will be used to obtain probabilities for each λ_i that will form the Final Prior.

Bayesian Expert Information Posterior. The Final Priors will be used to obtain posterior SPD's for each of the three phases of the CMGS Reliability Demonstration and for a combination of all three phases. Mission time will be assumed to be a standard three hour flight test time for use in calculating the reliability according to equation (1), $R_{CMGS} = \exp(-3*\lambda)$..

Step 5a. (Exponential) - The number of CMGS Reliability Demonstration test hours is adjusted by a series of k-factors to allow comparison to the standard mission time. This research accepts the use of the specified k-factors without examination. The number of failures is used with the total test time to calculate the posterior distribution based on the Final Prior distribution. This posterior distribution will be evaluated to determine a point estimate for λ and a Total Bayesian Probability Interval (TBPI).

Step 5b. (Binomial) - The binomial approach will focus on the number of successful missions that were accomplished out of the total number attempted. The Final Prior from Step 4.b above will be updated by this information to form the resulting posterior distribution. As in Step 5.a, both a point estimate and a TBPI will be determined for λ.

Bayesian Expert Information Point Estimate and Probability Interval. The resulting posterior distributions will be evaluated to obtain both point estimates for the failure rate, λ, and the 90% Total Bayesian Probability Interval (TBPI).

Step 6. The point estimate is the expected value of ,

$$E(\lambda) = \sum_i p(\lambda_i) * \lambda_i \tag{22}$$

and the TBPI is the interval that encompasses 90% of the area under the SPD while maximizing the sum of the probability densities associated with each point in the interval.

The methodology used to develop each of the six models will be applied to the CMGS Reliability Demonstration data and the results will be described in Chapter IV. The six model predictions will be compared with current Tomahawk flight test data to ascertain the accuracy of each model. An evaluation of both the approaches (Classical versus Bayesian) and the choice of distributions (exponential versus binomial versus expert information) will be based on 1) how the point estimators compare to the flight test mean value and 2) the width of the confidence or probability intervals for each of the models.

IV. Results

Overview

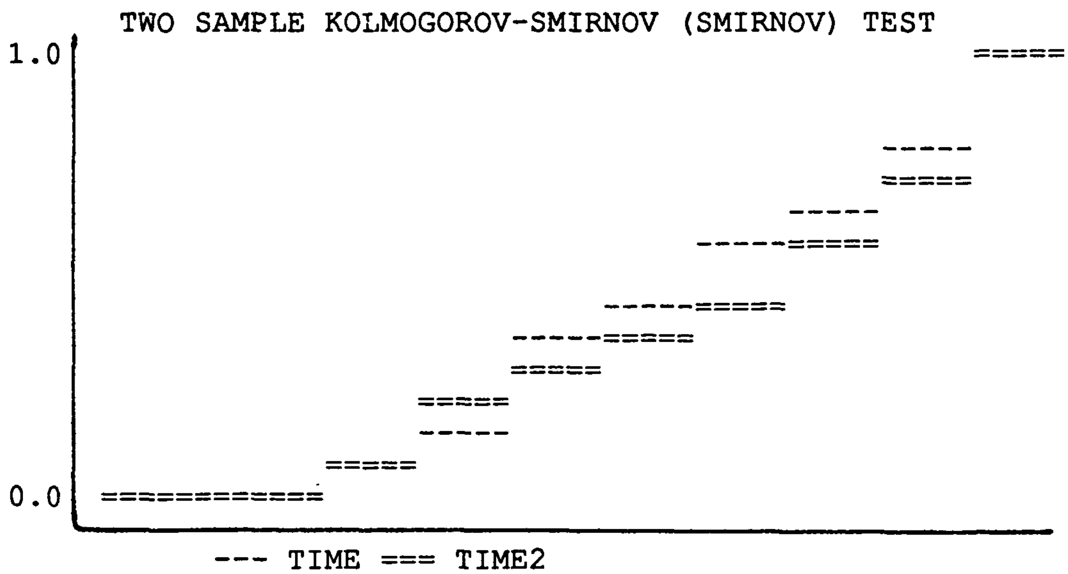
The results of the calculations described in the methodology of Chapter III will be described and discussed in this section. Most of the calculations will be shown in detail the first time presented and referenced in Appendix F for subsequent applications. The flight test data will be examined first to establish a benchmark with which to judge the prospective models. Point estimates and confidence intervals will be calculated in the order presented in the methodology, beginning with the classical models, continuing through the Bayesian distributions, and completing with the Bayesian expert information approach. This will be followed by a comparison of each of the model's predictions with the flight test values determined earlier. The flight test classical exponential and binomial approaches will be addressed next.

Flight Test

The flight test data (see Appendix A) was evaluated using the methods of classical statistical inference. The database (26) contained data from all 134 Tomahawk Land Attack Missile flight tests, representing a number of different configurations. In order to have enough data points to estimate the failure distribution, a Goodness-of-Fit test was conducted using failure data from the complete database. Reliability improvements were incorporated in some FY83 production units and in most FY84 and later units. Point estimates and confidence intervals were determined for three samples from the database: a) all flight tests, b) FY83 and on, and c) FY84 and on. Results of the Kolmogorov-Smirnov test will be reviewed next to see if the flight test failures were distributed exponentially.

Goodness-of-Fit Test. Using the sample point estimate for derived from the all of the flight test failures, an exponential distribution of random points was compared with the actual flight times. The results of the test are shown in Figure 6.

The p-value for this test indicates that the two distributions are not statistically distinct. This implies that the CMGS flight test failures are exponentially distributed. Because only two flight test failures have occurred with the FY83 and later configurations, all CMGS flight failures have been included for the Kolmogorov-Smirnov Test. Caution must be used in interpreting these results because of the small sample size and varying configurations of earlier flight test units. The point estimates and confidence intervals were calculated using all the flight test operating time, including those units that flew successfully. As such, the flight tests were regarded as a Type I, time truncated tests were directly comparable to the CMGS Reliability Demonstration results.




```
SAMPLE SIZE FOR TIME                                9
SAMPLE SIZE FOR TIME2                              11

HYPOTHESIS : TIME <> TIME2
TWO TAILED KOLMOGOROV-SMIRNOV STATISTIC          0.12

HYPOTHESIS : TIME < TIME2
ONE TAILED KOLMOGOROV-SMIRNOV STATISTIC          0.00

HYPOTHESIS : TIME > TIME2
ONE TAILED KOLMOGOROV-SMIRNOV STATISTIC          0.12
```

Figure 6. Flight Test Data Kolmogorov-Smirnov Test

The Classical Exponential Approach. The number of failures and total operating time for the flight tests is shown in Table 3. The point estimate for the failure rate is the number of failures divided by the total operating time in the test.

Classical Exponential Confidence Interval. The 90% confidence interval about the point estimate was obtained for the FY83 data using a chi square transformation with equation (11).

$$\text{Lower Limit} = \frac{1.635}{2 * 75.2} = .01087$$

$$\text{Upper Limit} = \frac{9.488}{2 * 75.2} = .06309$$

The Binomial Approach. This approach used the number of successful missions completed divided by the number attempted to develop a point estimate for p, the probability of success. This value was then converted to a failure rate using the exponential relationship described in equation (1). The binomial point estimates and confidence intervals for the flight test data are shown in Table 3.

Classical Binomial Confidence Interval. The 90 % confidence interval about the point estimate was approximated using a transformation to the F distribution as described in equation (12). For the flight test data,

Table 3. Flight Test Results

Exponential					
Data	Time (T)	Failures (R)	$\hat{\lambda}$ (R/T)	Confidence Limit Lower	Upper
All	176.441	12	.06801	.04358	.10321
FY83	75.2	2	.02660	.01087	.06309
FY84	54.65	1*	.01830	.006502	.05481

Binomial						
Data	Number of Missions Success (X)	Attempt (N)	P (X/N)	$\hat{\lambda}$	Confidence Limit Lower	Upper
All	107	119	.8992	.03543	.02301	.05253
FY83	48	50	.9600	.01361	.003657	.03610
FY84	41	42*	.9762	.008033	.0007849	.03118

* One failure assumed to occur on next mission, t = 0

$$\text{Lower Limit} = \frac{(48+1)*F_{.95}(2*48+2,\ 2*50-2*48)}{50-48+(48+1)*F_{.95}(2*48+2,\ 2*50-2*48)}$$

$$= \frac{49*F_{.95}(98,\ 4)}{2+49*F_{.95}(98,\ 4)} = \frac{49*3.7}{2+49*3.7} = .9891$$

and converting to a failure rate, R(t) = exp(-λ*3),

$$= \ln(.9891)/-3 = .003657$$

$$\text{Upper Limit} = \frac{48}{48+(50-48+1)*F_{.95}(2*50-2*48+2,\ 2*48)}$$

$$= \frac{48}{48+3*F_{.95}(6,96)} = \frac{48}{48+3*1.83} = .8974$$

$$= \ln(R)/(-3) = \ln(.8974)/-3 = .03610$$

The differences in the flight test λ point estimates and confidence intervals between the exponential and binomial approaches raise concerns over the use of this flight test data as a benchmark for model performance. If the actual failure rate λ was exponentially distributed, then based on the model mission time, the binomial and exponential values should have been equivalent.

The configuration of the CMGS units had an impact on the demonstrated flight test reliability. The FY84 configuration was probably most representative of the current CMGS configuration, however, because the point estimates and confidence intervals for the failure rate were derived by including a hypothetical failure, the true λ for this population was probably lower than indicated above. Lacking sufficient data for the FY84 configuration, the FY83 configuration data was used as a prior for the Bayesian approaches.

The Classical Models

As described in the methodology, both exponential and binomial approaches were used with the CMGS Reliability Demonstration data (28-30) from each of the phases of testing. A prediction for the point estimate and a corresponding confidence interval was made for a combination of all of the data as well.

<u>The Classical Exponential Approach</u>. The number of failures and total operating time in each test is shown in Table 4 for each of the test phases. The point estimate for the failure rate λ was the number of failures divided by the total operating time in the test.

<u>Classical Exponential Confidence Interval.</u> The 90% confidence interval about the point estimate was obtained using a chi square transformation with equation (11).

$$\text{Lower Limit} = \frac{2.733}{2 * 946.846} = .001444$$

$$\text{Upper Limit} = \frac{12.592}{2 * 946.846} = .006648$$

Table 4. Classical Exponential Model

Test	Time (T)	Failures (R)	λ (R/T)	Confidence Limit Lower	Confidence Limit Upper
Phase I	946.486	3	.003170	.001444	.006648
Phase II	423.11	1	.002364	.0008399	.007080
Phase I Extension	881.413	3	.003404	.001550	.007142
Combined	2251	7	.003110	.001769	.005260

The Binomial Approach. This approach used the number of successful missions completed divided by the number attempted to develop a point estimate for p, the probability of success. This value was then converted to a failure rate using the exponential relationship described in equation (1). The data and resulting values for the CMGS Reliability Demonstration test phases are shown in Table 5.

Table 5. Classical Binomial Model

Test	Number of Success (X)	Missions Attempt (N)	X/N	λ	Confidence Limit Lower	Upper
Phase I	279	282	.9894	.003565	.001310	.007893
Phase II	125	126	.9921	.002656	.0002787	.01034
Phase I Extension	261	264	.9886	.003810	.00140	.008424
Combined	665	672	.9896	.003490	.001941	.005922

Classical Binomial Confidence Interval. The 90 % confidence interval about the point estimate was approximated using a transformation to the F distribution as described in equation (12).

Classical Binomial Confidence Interval. The confidence interval was determined using equation (12) and the F distribution for 1- /2 = .95. For the Phase I data,

$$\text{Lower Limit} = \frac{(279+1)*F_{.95}(2*279+2,\ 2*282-2*279)}{282-279+(279+1)*F_{.95}(2*279+2,\ 2*282-2*279)}$$

$$= \frac{280*F_{.95}(560,\ 6)}{3+280*F_{.95}(560,\ 6)} = \frac{280*2.72}{3+280*2.72} = .9961$$

and converting to a failure rate, $R(t) = \exp(-\lambda*3)$,

$$= \ln(.9961)/-3 = .001310$$

$$\text{Upper Limit} = \frac{279}{279+(282-279+1)*F_{.95}(2*282-2*279+2,\ 2*279)}$$

$$= \frac{279}{(279+4*F_{.95}(8,558)} = \frac{279}{279+4*1.68} = .9765$$

$$= \ln(R)/(-3) = \ln(.9765)/-3 = .007934$$

There was good agreement between the classical exponential and binomial point estimates for λ. In both cases, they were considerably lower than the flight test failure rates, and had confidence intervals enclosing the point estimates that were an order of magnitude smaller than the flight test confidence intervals. The binomial confidence intervals were wider than those for the exponential approach.

The Bayesian Models

The next sections describe the results obtained from the Bayesian exponential, binomial and expert information models, the last having both exponential and binomial approaches. In the development of each model, a sequence of steps was followed:

1) Develop prior distribution,

2) Calculate posterior distribution or iterative priors,

3) Determine Bayesian point estimate for failure rate,

4) Determine Total Bayesian Probability Interval.

The Bayesian exponential model was examined first.

Bayesian Exponential Model. This model used flight data from comparable CMGS configurations to develop the gamma(α,β) prior distribution. The prior was then updated by results from the CMGS Reliability Demonstration to predict point estimates and TBPI's for each of the three test phases and the combination of all three.

Bayesian Exponential Prior. For the purposes of this research, flight test data from CMGS units built in FY83 and later production lots was used to develop the prior distribution. As described in the methodology, the simplest estimate for the α and β parameters of the gamma distribution were the number of flight test failures, s, and total test time, t. The gamma(2,75.2) prior distribution, based on the flight test data, was used to obtain the posterior distribution.

Bayesian Exponential Preposterior Analysis. The prior was evaluated using two hypothetical samples of data. The likely data was one failure in 256 hours and resulted in the following point estimate:

$$\frac{s+\alpha}{t+\beta} = \frac{1+2}{256+75.2} = .009058$$

The unlikely data was ten failures in the same 256 hours for a point estimate of

$$\frac{s+\alpha}{t+\beta} = \frac{10+2}{256+75.2} = .03623$$

These values indicated that the gamma(2,75.2) prior was performing in a proper manner. If the point estimates for these two data samples had been similar, the prior would have been adjusted.

Bayesian Exponential Posteriors. The gamma(2,75.2) prior was updated by the CMGS Reliability Demonstration failures, s, and total time, t, in test for each of the three phases and the combination of all three. The values for s and t are tabulated in Table 6 along with the point estimates and TBPI's for the new gamma($s+\alpha$,$t+\beta$) posterior distributions.

Bayesian Exponential Point Estimate. Equation (16) defines the Bayesian point estimator for the mean. The calculation for Phase I data was

$$\frac{s+\alpha}{t+\beta} = \frac{3+2}{946.486+75.2} = .004894$$

<u>Bayesian Exponential Probability Interval</u>. The TBPI for each of the CMGS Reliability Demonstration test phases was calculated using chi square percentile values. Phase I calculations are shown below, based on equation (17), and the remaining values are tabulated in Table 6.

$$\text{Lower Limit} = \frac{X^2_{.95}(2*3 + 2*2)}{2*(946.486 + 75.2)} = \frac{3.94}{2043.37} = .001928$$

Table 6. Bayesian Exponential Model

Test	Time (T)	# of Failures (R)	$\frac{s+\alpha}{t+\beta}$	Confidence Limit Lower	Confidence Limit Upper
Phase I	946.486	3	.004894	.001928	.008961
Phase II	423.11	1	.006020	.001641	.01263
Phase I Extension	881.413	3	.005338	.002106	.009775
Combined	2251	7	.003869	.002018	.006205

$$\text{Upper Limit} = \frac{X^2_{.05}(2*3 + 2*2)}{2*(946.486 + 75.2)} = \frac{18.31}{2043.37} = .008961$$

<u>Bayesian Binomial Model</u>. For the binomial approach, the $\beta(x,n)$ distribution was used for both the prior and posterior distributions. X was the number of successful missions accomplished and n was the number attempted. As in the Bayesian exponential model, the prior was based on

relevant flight test data and updated with the results of the CMGS Reliability Demonstration.

<u>Bayesian Binomial Prior</u>. The $\beta(x_0, n_0)$ distribution was used for the prior, where x_0 was the number of flight test successes and n_0 was the total number of flights attempted. As in the Bayesian exponential model, only flights using CMGS units produced in FY83 and later production lots were scored.

<u>Bayesian Binomial Preposterior Analysis</u>. The preposterior analysis for this approach used binomial hypothetical data. The likely data was 99 successes out of 100 attempts with a point estimate of:

$$\frac{x + x_0}{n + n_0} = \frac{99 + 48}{100 + 50} = .9800$$

convert to λ, $\lambda = \ln(.9800)/-3 = .006734$

The unlikely data was 90 successes out of 100 attempts with a point estimate of:

$$\frac{x + x_0}{n + n_0} = \frac{90 + 48}{100 + 50} = .9200$$

convert to λ, $\lambda = \ln(.9200)/-3 = .02779$

The results of this analysis again indicated that the prior was functioning properly, as evidenced by the wide difference between the two failure rates.

Bayesian Binomial Posterior. The $\beta(48,50)$ prior was updated with the results of the CMGS Reliability Demonstration to form four new posterior $\beta(x+x_0, n+n_0)$ distributions (including the combination of phase data). The x and n parameters were the successes and total attempts for each of the test phases.

Bayesian Binomial Point Estimate. The point estimator for the reliability was calculated for Phase I data using equation (19). The results of point estimate and TBPI calculations for other phases are shown in Table 7.

$$\frac{x + x_0}{n + n_0} = \frac{279 + 48}{282 + 50} = .9849$$

convert to λ, $\lambda = \ln(.9849)/-3 = .005058$

Table 7. Bayesian Binomial Model

Test	Number of Success (X)	Missions Attempt (N)	$\frac{x+x_0}{n+n_0}$	λ	Confidence Limit Lower	Confidence Limit Upper
Phase I	279	282	.9849	.005058	.002465	.008082
Phase II	125	126	.9830	.005731	.002115	.01016
Phase I Extension	261	264	.9841	.005351	.002608	.008615
Combined	665	672	.9875	.004181	.002525	.006038

Bayesian Binomial Probability Interval. The TBPI for the $\beta(x+x_0, n+n_0)$ posterior distribution given by equation (20) was calculated for Phase I data as shown below. Other phases are included in Table 7.

$$\text{Lower Limit} = \frac{(279+48)*F.95(2*279+2*48,2*282+2*50-2*279-2*48)}{282+50-279-48+(279+48)*F.95(2*279+2*48,\ 2*282+2*50-2*279-2*48)}$$

$$= \frac{327*F.95(654,10)}{5+327*F.95(654,10)} = \frac{327*2.06}{5+327*2.06} = .9926$$

to convert to λ, ln(.9925)/-3 = .002465

$$\text{Upper Limit} = \frac{279 + 48}{279+48+(282+50-279-48)*F.95(2*282+2*50-2*279-2*48,\ 2*279+2*48)}$$

$$= \frac{327}{327+5*F.95(10,654)} = \frac{327}{327+5*1.605} = .9761$$

to convert to λ, ln(.9761)/-3 = .008082

Analysis. These Bayesian models had point estimates for each of the individual test phases that were consistently higher than the classical estimates. Those for the combined values were much closer, indicating the sensitivity of the posterior to the size of the sample. Again, all of these point estimates were significantly lower than the flight test results. In contrast to the classical values, the confidence intervals for the Bayesian exponential approach were wider for each of the four test cases than those of the Bayesian binomial approach, though not by an appreciable amount.

Bayesian Expert Information Model. The expert information model made use of the judgement of weapon system experts to construct Subjective Probability Distributions (SPD) that reflect the relative uncertainty of their judgements. These SPD's were then modified by the Bayesian process to incorporate new information and used as management tools for decision making. This research used a modification of the Kaplan model explained in Chapters II & III to obtain a point estimate and probability interval for the failure rate of the CMGS based on the steps outlined in the methodology.

Bayesian Expert Information Prior. The Final Prior was the product of a series of iterative updates with each successive posterior distribution being the next prior distribution. The calculations for each update are provided in Table 8, and a sample calculation is demonstrated for the generation of the Second Prior from the First Prior.

Step 1. The specified MTBF for the CMGS was 256 hours (29:1D), which corresponded to a failure rate of .003906. Using a three hour standard mission, this failure rate had a reliability of .9884. In addition to this specified value, there were predicted values for the CMGS reliability based on parts counts, etc.. The predicted reliability of the RMUC (9:3-70) was .9905, with a failure rate of .003182, and it may be regarded as an upper boundary of the CMGS reliability envelope. The predicted CMGS reliability was .9868, slightly less than the above specified value, with an associated failure rate of .004429. It was considered unlikely that the failure rates of the CMGS would deviate greatly from these values. In order to account for the variance, the range of values for the First Prior was expanded to .001 through .006. Given the range, it was necessary to attach a measure of the uncertainty inherent in value.

Expert information was required to evaluate the risk or uncertainty. The SPD selected to represent the predicted reliability and associated failure rates is plotted in Figure 9; it shows higher confidence in the central values of the range. This First Prior was based on the subjective determination of available information by the informed source.

Step 2. The specified MTBF requirement (35:6) for a similar component, the ALCM INE, was 395 hours. The demonstrated MTBF was 424 hours in the period from 30 November 1981 through 31 December 1986, involving approximately 576 failures in 91,114 hours for 1,687 ALCM INE units (see Appendix B). The 29 hour increase represented a 7.3418% increase over the specified MTBF. It should be noted that these units were built during FY83 and FY84 production lots, the same production lots used for the

Table 8. Bayesian Expert Information First through Third Priors

First Prior = SPD based on CMGS Specification MTBF = 256 hrs

λi	0.001	0.002	0.003	0.004	0.005	0.006
p1(λi)	0.1	0.15	0.25	0.25	0.15	0.1

Second Prior = First Prior updated with ALCM INE 6.842% decrease.

λi	0.001	0.002	0.003	0.004	0.005	0.006
p1(λi)	0.1	0.15	0.25	0.25	0.15	0.1
R=exp(-3) =	0.9970	0.9940	0.9910	0.9880	0.9851	0.9821
R+.0007925 =	0.9977	0.9948	0.9918	0.9888	0.9859	0.9829
λ(adj.)	7.35E-4	0.001734	0.002734	0.003733	0.004732	0.00573
(.3/.7)	0.145	0.18	0.25	0.22	0.135	0.07
p1(λi)p(E|λi)=	0.0145	0.027	0.0625	0.055	0.02025	0.007
/Σ(p(E|λi) =						
p2(λi)=	0.07785	0.1449	0.3355	0.2953	0.1087	0.03758

Third Prior = Second Prior updated with FCII reliability decrease .005205

R=exp(-3)-.005205	0.9917	0.9888	0.9858	0.9828	0.9799	0.9769
λi	0.002745	0.00375	0.004755	0.005761	0.006766	0.00777
(.25/.75)	0.01946	0.09463	0.1926	0.3255	0.2486	0.1191
λi(adj)	0.001	0.002	0.003	0.004	0.005	0.006
p2(λi)	0.07785	0.1449	0.3355	0.2953	0.1087	0.0375
p(E|λi)	0.01946	0.09463	0.1926	0.3255	0.2486	0.1191
p2(λi)*p(E|λi)=	0.001515	0.01371	0.06463	0.09612	0.02703	0.00447
/Σ(p(E|λi)						
p3(λi)=	0.007302	0.06611	0.3114	0.4632	0.1302	0.0215

flight database calculations on the CMGS units. The MTBF increase corresponded to a 6.8414 % decrease in the failure rate λ. This, in turn, resulted in an increase in the reliability of the CMGS units, calculated over a standard three hour mission. There was a shift down in the failure rate of approximately .0003 for each .001 division of λ.

For example, the previous probability density for λ = .006 was .10. With the shift, λ = .005731, leaving 70 % of the original probability density with the .006 category, and transferring 30% to the .005 category. This continued to the .001 category, which kept its .10 and the additional 30% from category .002 for a grand total of .1450. Using Bayes' Theorem, it was straightforward to perform the indicated mathematical operations and normalize the function so the area under the SPD was equal to 1.0. The resulting SPD is the Second Prior.

The Second Prior had greater probability density concentrated in the central values of the λ range. Thus, the effect of the ALCM INE data was to both reinforce the original central tendency and shift it toward lower values.

Step 3. The design weaknesses identified in the Ground Launched Cruise Missile Flight Critical Item Investigation (11:2.1-4.16) were used to update the Second Prior. Table 9 tabulates the weaknesses and their effect on the CMGS reliability. These values reduced the reliability estimates and increase the probability of observing higher failure rates. The SPD for λ was therefore adjusted by shifting it toward higher failure rates by an amount corresponding to the reduced reliability, .0052050. This factor was the summation of risk values tied to critical failure modes

Table 9. GLCM WSAP Flight Critical Item Design Weaknesses

Failure Modes	(FCI's)	Component	Failure Probability	Reliability Impact
41,228	(2824)	Missile	-	-
36,272	(2280)	CMGS	-	-
33,363	(2157)	RMUC	-	-
Not Tested	855	"	> 1 x 10-6	8.55 x 10-4
"	31	Other	> 5 x 10-5	1.55 x 10-3
"	28	CMGS	> 1 x 10-4	2.8 x 10-3
Total	916			5.205 x 10-3

that were not tested adequately. "Inadequately tested" was a term defined by the GLCM WSAP FCII based on the level of testing performed on each component, the criticality of the failure modes and probability of failure.

The decreased reliability of .005205 was subtracted from the calculated reliability values for each λ_i and a new λ_j was obtained for the resulting decreased reliability. For example, the new λ_j for λ_i = .001 was ln(.997004-.005205)/-3, which equaled .002745. Because this value was about 75% across the interval between .002 and .003, the curve was shifted to the right by the same 75 %. Thus, 25 % of the probability density for .001 remained at .001. The remaining 75% of .077852 + 25% of .144966, the probability density for .002, made up the new probability density for .002, or .094607.

These new probability values were used to update the Second Prior for the design weaknesses -- the resulting posterior was the Third Prior. As in Step 2 above, the central tendency was reinforced, but with a shift toward higher values. Although the ALCM INE and FCII updates might have reasonably been expected to cancel their respective effects, there was a new combined probability density of 77.5% for $.003 \leq \lambda \leq .004$ versus the First Prior probability density of 50% for the same values.

Step 4. The Environmental Stress Screening and acceptance testing results (32-34) were not suitable for inclusion in this analysis. Examination of the data indicated that the CMGS component screening process was allowing an unacceptable number of quality escapes and infant mortality to pass on to the missile acceptance testing. Additionally, the missile Acceptance Test Procedure had ESS and functional checkouts intermingled to an extent that it was not possible to extract meaningful data for either. Finally, data was not available on the number of attempts necessary to completely pass through all the tests. Although there was first pass yield data available, because of the problems with test structure and screening levels referenced above, this data was not suitable for the research (see Appendix C). This step was bypassed.

Step 5a. (Exponential) -- Flight test data was used to update the Third Prior and form the posterior that was the Final Prior. For the exponential approach, equation (21) was used with n failures in time T. Considering flight test data (26) for CMGS units produced in FY83 and later, n = 2 and T = 75.2. Again using the standard three hour mission, it was possible to calculate the probability, $p(E|\lambda i)$, of two failures in 75.2 hours, given each of the failure rates, λi. In the Third Prior, for $1 \simeq .001$, $p(E|\lambda 1) = .002623$, and the corresponding Final Prior value was .00063. Other values are tabulated in Table 10.

The Final Prior was ready to use for calculating exponential posterior distributions for each of the phases of the CMGS Reliability Demonstration. Before proceeding, the Final Prior needed to be determined for the binomial approach.

Step 5b. (Binomial) -- The Final Prior was calculated by updating the Third Prior with the results of flight testing of the current configuration, 48 successful out of

Table 10. Bayesian Expert Information
Exponential Final Prior

Final Prior = Third prior updated by Flight Test times for FY83 and on. 2 failures, 75.2 hours.

λi	0.001	0.002	0.003	0.004	0.005	0.006
$p3(\lambda i)$	0.007302	0.06611	0.3114	0.4632	0.1302	0.02157
$p(E\|\lambda i)$	0.002623	0.009731	0.02031	0.03349	0.04854	0.06483
$p3(\lambda i)*p(E\|\lambda i)=$	1.90E-05	0.000643	0.006326	0.01551	0.006323	0.00139
$/\Sigma(p(E\|\lambda i)$ pF$(\lambda i)=$	0.00063	0.02129	0.2093	0.5133	0.2092	0.04628

50 attempts. Equation (21) was used in the binomial format and the calculations were made in a similar fashion as in the exponential approach once $P(E|\lambda i)$ had been determined. For 1 = .001, $P(E|\lambda i)$ = .000867 and the resulting posterior probability was .002257. Other values are tabulated in Table 11.

Table 11. Bayesian Expert Information Binomial Final Prior

Final Prior = Third Prior updated by Flight Test results using FY83 & on, 48/50

λi	0.001	0.002	0.003	0.004	0.005	0.006
p3(λi)	0.007302	0.06611	0.3114	0.4632	0.1302	0.0215
p(E|λi)	0.001168	0.004451	0.009544	0.01617	0.02408	0.0330
p3(λi)*p(E|λi)=	8.53E-06	0.00029	0.002973	0.00749	0.003137	0.00071
\Σ(p(E|λi)						
pF(λi)=	0.000583	0.02013	0.2034	0.5125	0.2146	0.0487

Both the exponential and binomial approaches resulted in similar Final Priors. The chief impact of the flight test data was to further centralize the probability density and simultaneously shift it toward slightly higher values of λ.

Bayesian Expert Information Posteriors. The Final Priors were used with the results of the CMGS Reliability Demonstration to calculate the Posteriors for each of the three test phases and the combination of all three. The calculations were based on equation (21) in the same manner as in Steps 4.a and 5.b above. Exponential results are tabulated in Table 12; binomial results are in Table 13.

Bayesian Expert Information Probability Intervals. The TBPI was calculated using an α value of .1, corresponding to a 90 % confidence limit. The total area under the SPD was equal to one. The 90 % TBPI was defined by the two endpoints of the shortest interval that encompassed 90% of the area under the distribution. To calculate the interval, ten increments were spaced between each λ value and the area was set equal to .90. An estimate of the interval length was made and moved along the λ x-axis incrementally. The shortest interval that still encompassed an area of .90 was the TBPI. The point estimates and TBPI endpoints are shown in Table 14.

Analysis. The expert information exponential and binomial approaches were remarkably consistent. The point estimates were in the range of values for the other models, and were significantly lower than the flight test results. The real difference in this approach was the noticeably narrower confidence intervals. In every case, the width did not exceed .003, less than any other interval calculated in this research.

A summary of the results will be presented in Chapter V. The potential applications of this research will be discussed, along with suggestions for future investigations.

Table 12. Bayesian Expert Information
Exponential Posteriors

First Case - Exponential Phase I 946.486 hours, 3 failures

λi	0.001	0.002	0.003	0.004	0.005	0.006
pF(λi)	0.00063	0.02129	0.2093	0.5133	0.2092	0.0462
p(E|λi)	0.05485	0.1703	0.2230	0.2052	0.1555	0.1043
pF(λi)*p(E|λi)/Σ(p(E|λi)=	0.00018	0.01878	0.2419	0.5456	0.1686	0.02501

Point Estimate = 0.003939

Second Case - Exponential Phase II, 423.11 hours, 1 failure

λi	0.001	0.002	0.003	0.004	0.005	0.006
pF(λi)	0.00063	0.02129	0.2093	0.5133	0.2092	0.04628
p(E|λi)	0.2771	0.3631	0.3567	0.3115	0.2551	0.2005
pF(λi)*p(E|λi)/Σ(p(E|λi)=	0.000576	0.02533	0.2447	0.5241	0.1749	0.03041

Point Estimate = 0.003939

Third Case - Exponential Phase I Extension, 881.4128 hours, 3 failures

λi	0.001	0.002	0.003	0.004	0.005	0.006
pF(λi)	0.00063	0.02129	0.2093	0.5133	0.2092	0.04628
p(E|λi)	0.04727	0.1566	0.2189	0.2149	0.1739	0.1244
pF(λi)*p(E|λi)/Σ(p(E|λi)=	0.000149	0.01653	0.2272	0.5471	0.1804	0.02856

Point Estimate = 0.003977

Fourth Case - Exponential Combined 2251 hours, 7 failures

λi	0.001	0.002	0.003	0.004	0.005	0.006
pF(λi)	0.00063	0.02129	0.2093	0.5133	0.2092	0.04626
p(E|λi)	0.006118	0.08245	0.1483	0.1170	0.05874	0.02216
pF(λi)*p(E|λi)/Σ(p(E|λi)=	3.65E-05	0.01653	0.2924	0.5656	0.1158	0.00966

Point Estimate = 0.00381

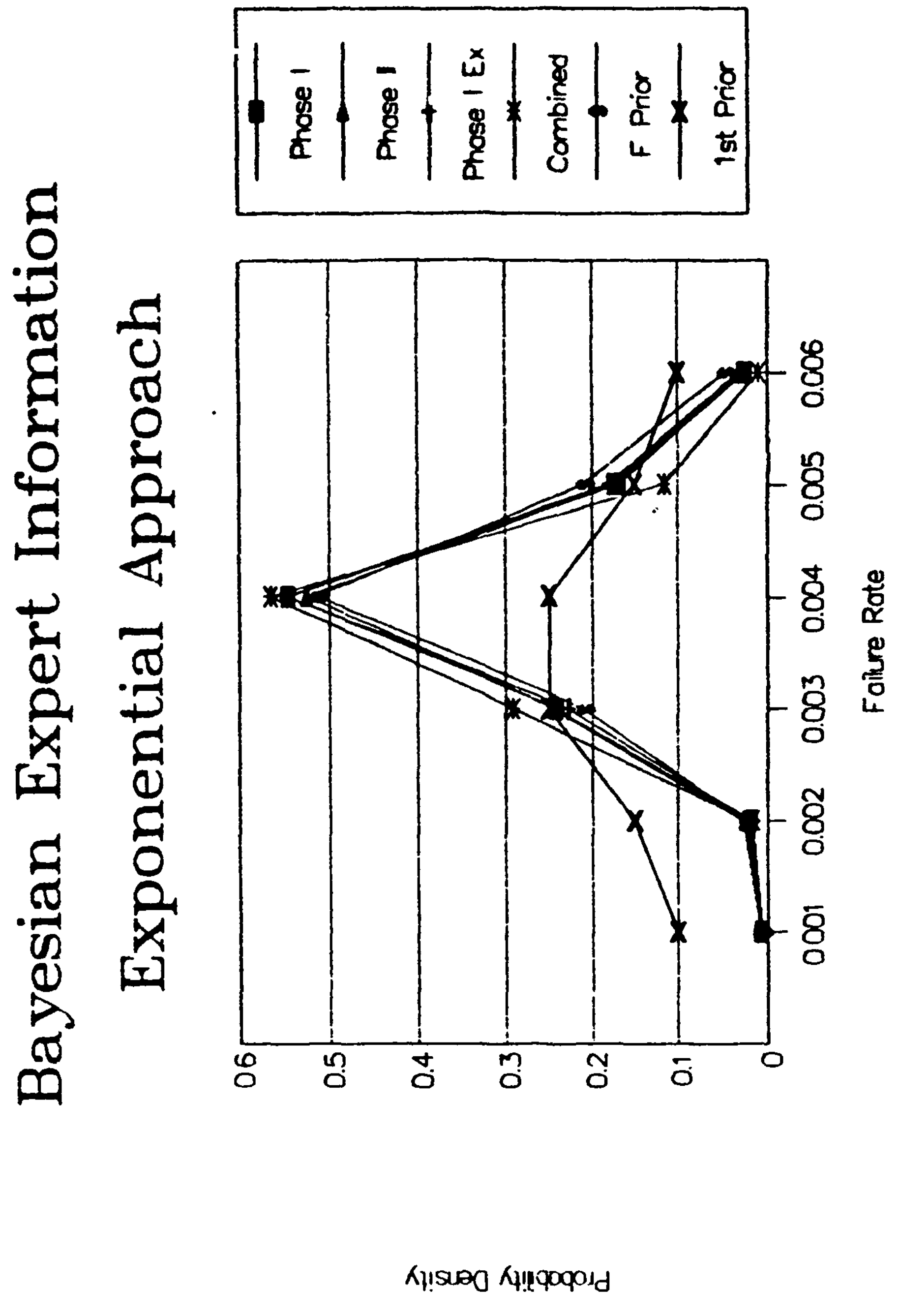


Figure 7. Bayesian Expert Information
Exponential Approach

Table 13. Bayesian Expert Information Binomial Posteriors

First Case - Binomial Phase I 279/282

λi	0.001	0.002	0.003	0.004	0.005	0.006
$pF(\lambda i)$	0.000583	0.02013	0.2034	0.5125	0.2146	0.04877
$p(\varepsilon \mid \lambda i)$	0.002797	0.01692	0.04317	0.07735	0.1141	0.1490
$pF(\lambda i)*p(\varepsilon \mid \lambda i)/\Sigma(p(\varepsilon \mid \lambda i) =$	2.03E-05	0.00423	0.1091	0.4922	0.3042	0.09023

Point Estimate = 0.004367

Second Case - Binomial Phase II 125/126

λi	0.001	0.002	0.003	0.004	0.005	0.006
$pF(\lambda i)$	0.000583	0.02013	0.2034	0.5125	0.2146	0.04877
$p(\varepsilon \mid \lambda i)$	0.1111	0.1962	0.2596	0.3053	0.3366	0.3563
$pF(\lambda i)*p(\varepsilon \mid \lambda i)/\Sigma(p(\varepsilon \mid \lambda i) =$	0.000214	0.01304	0.1743	0.5166	0.2385	0.0573

Point Estimate = 0.004152

Third Case - Binomial Phase I extension 261/264

λi	0.001	0.002	0.003	0.004	0.005	0.006
$pF(\lambda i)$	0.000583	0.02013	0.2034	0.5125	0.2146	0.04877
$p(\varepsilon \mid \lambda i)$	0.002335	0.01438	0.03736	0.06816	0.1024	0.1361
$pF(\lambda i)*p(\varepsilon \mid \lambda i)/\Sigma(p(\varepsilon \mid \lambda i) =$	1.91E-05	0.004053	0.1064	0.4889	0.3077	0.09293

Point Estimate = 0.004379

Fourth Case - Binomial Combined 665/672

λi	0.001	0.002	0.003	0.004	0.005	0.006
$pF(\lambda i)$	0.000583	0.02013	0.2034	0.5125	0.2146	0.04877
$p(\varepsilon \mid \lambda i)$	6.12E-06	0.000402	0.003529	0.01356	0.03316	0.0608
$pF(\lambda i)*p(\varepsilon \mid \lambda i)/\Sigma(p(\varepsilon \mid \lambda i) =$	2.01E-07	0.000456	0.04041	0.3913	0.4007	0.1672

Point Estimate 0.004694

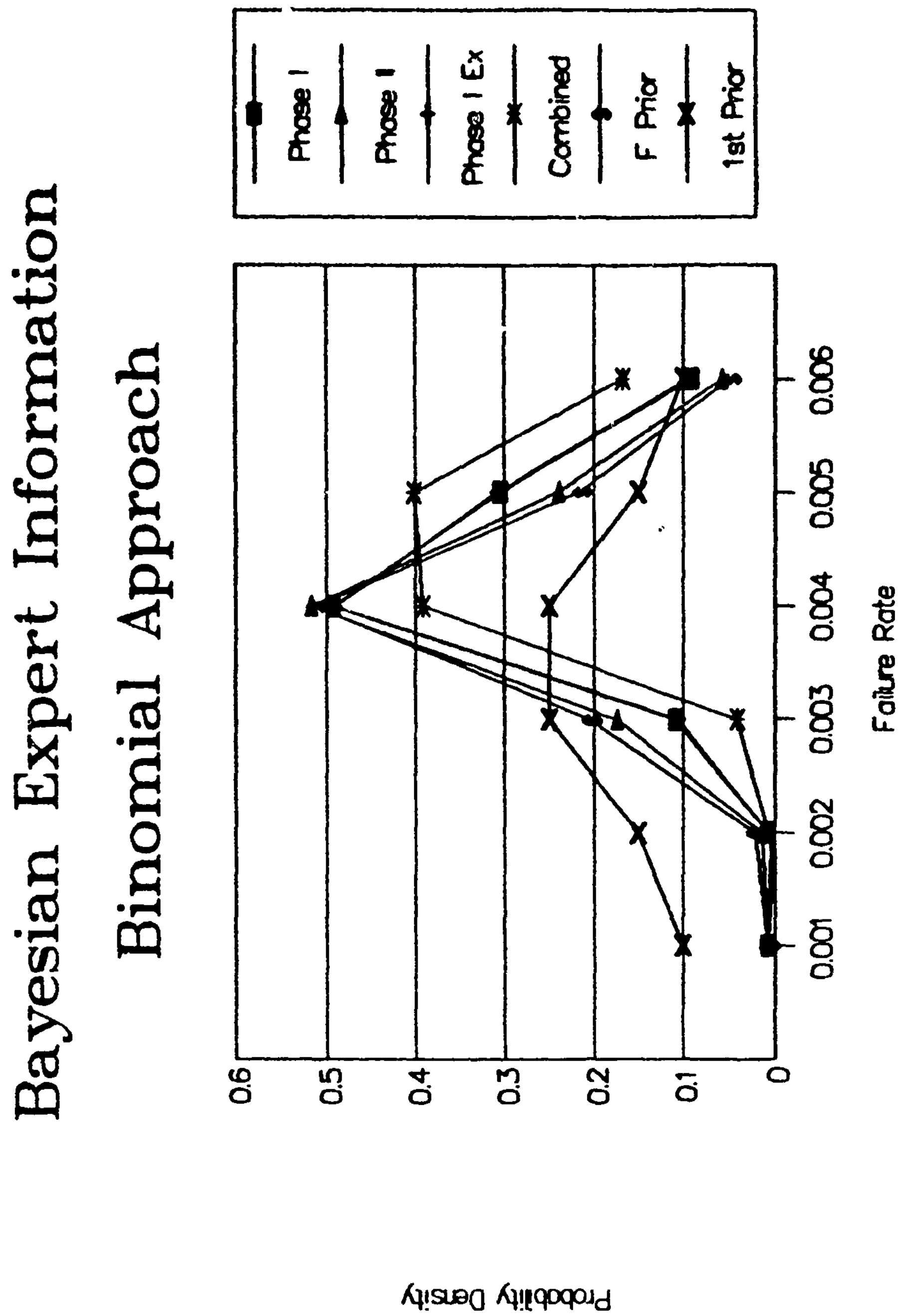


Figure 8. Bayesian Expert Information Binomial Approach

Table 14. Bayesian Expert Information Probability Intervals

Test Phase	Point Estimate	Probability Lower	Upper	Interval Confidence
Exponential Approach:				
Phase I	.003946	.0025	.0051	.9055
Phase II	.003947	.0025	.0052	.9087
Phase I Extension	.003984	.0025	.0051	.9006
Combined Phases	.003815	.0025	.0048	.9043
Binomial Approach:				
Phase I	.004372	.0025	.0054	.9055
Phase II	.004157	.0026	.0054	.9116
Phase I Extension	.004384	.0025	.0054	.9030
Combined Phases	.004698	.0035	.0061	.9090

V. Recommendations

Data Analysis

A summary of the point estimates for λ and the width of 90% confidence intervals is displayed in Table 15 for each of the models and CMGS test phases. The military manager who is facing a weapon systems decision will often request reliability analyses be performed to determine the most likely reliability point estimate. It is often difficult to make the decision without knowing something about the uncertainty or risk associated with accepting that point estimate. The savvy military manager will also request an analysis be performed to determine the confidence interval surrounding the point estimate. As was explained earlier in Chapter 2, the classical confidence interval is stating that for a certain percentage of trials, the true population value will fall within a range about the point estimate. The Total Bayesian Probability Interval is stating the probability that the true value is within a range about the point estimate for each trial. The common interpretation of each of these is that they represent the risk assumed by the manager in accepting the point estimate as the true value. If the intervals are very narrow, the manager believes that even if the point estimate is not correct, it is very close to the actual value. Similarly, if the interval is wide, the true value could take on a range of values, some of

Table 15. Point Estimates and Confidence Interval Widths

Phase I	Classic Expon	.003170	.005204
Phase I	Classic Binomial	.003565	.006583
Phase I	Bayes Expon	.004894	.007033
Phase I	Bayes Binomial	.005058	.005617
Phase I	Expert Expon	.003939	.0026
Phase I	Expert Binomial	.004367	.0029
Phase II	Classic Expon	.002364	.0062401
Phase II	Classic Binomial	.002656	.0100613
Phase II	Bayes Expon	.006020	.01099
Phase II	Bayes Binomial	.005731	.008045
Phase II	Expert Expon	.003939	.0027
Phase II	Expert Binomial	.004152	.0028
Phase I Ex	Classic Expon	.003404	.005592
Phase I Ex	Classic Binomial	.003810	.007024
Phase I Ex	Bayes Expon	.005338	.007669
Phase I Ex	Bayes Binomial	.005351	.006007
Phase I Ex	Expert Expon	.003977	.0026
Phase I Ex	Expert Binomial	.004379	.0029
Combined	Classic Expon	.003110	.003491
Combined	Classic Binomial	.003490	.003981
Combined	Bayes Expon	.003869	.004187
Combined	Bayes Binomial	.004181	.003513
Combined	Expert Expon	.003810	.0023
Combined	Expert Binomial	.004694	.0026

which might be quite different from the point estimate. Therefore, for the purposes of managerial decision-making, the shorter the confidence interval surrounding the point estimate, the lower the risk in accepting the point estimate. The discussion of these results and a comparison with the flight test point values follows.

Models. Within each of the different model approaches, relatively consistent point estimates for λ were obtained for each of the test phases and combined phases data. For example, Table 15 shows the classical exponential point estimates for Phase I, II, I-extension, and combined phases data are .003170, .002364, .003404, and .003110 .

There is a similar trend among the confidence interval widths through the three test phases, .005204, .006240, and .005592 for the classical exponential model, with relatively consistent widths for each model approach. The confidence interval widths for the combined phases data, .003491 for the classical exponential model, are shorter for all the approaches, predictably the result given the greater amount of data used in those calculations.

The point estimates for the classical approaches were always the smallest values, followed by the expert information and then Bayesian approaches. This reflects the relative effect of flight test data in the prior distributions. For example, the classical approach only considers the CMGS Reliability Demonstration data; the flight test data has no effect. Hence, this approach would show the greatest variance from the flight test point estimates.

The expert information approach confidence interval widths were the smallest in every case. The classical approach intervals were slightly smaller than those of the Bayesian approach, but still large compared to the expert information values.

Flight Test Results. The CMGS flight test failures were concluded to be distributed exponentially when examined over the total flight test program. It was not possible to conduct a meaningful Goodness-of-Fit test for the flight test failures of CMGS units with comparable configurations to those used in the CMGS Reliability Demonstration because of the limited number of failures. A Weibull plot of the total failures shown in Figure 9 indicates that the shape parameter β was approximately 2.0, significantly above the 0.0 to 1.0 range associated with a strictly exponential failure rate. This is also supported by the differences between exponential and binomial approaches for each of the models. It appears that there may be other failure mechanisms at work based on the limited data available.

Because the CMGS configuration changes that were intended to improve reliability were introduced over a period of time, it would be reasorable to expect a range of reliability characteristics corresponding to the different

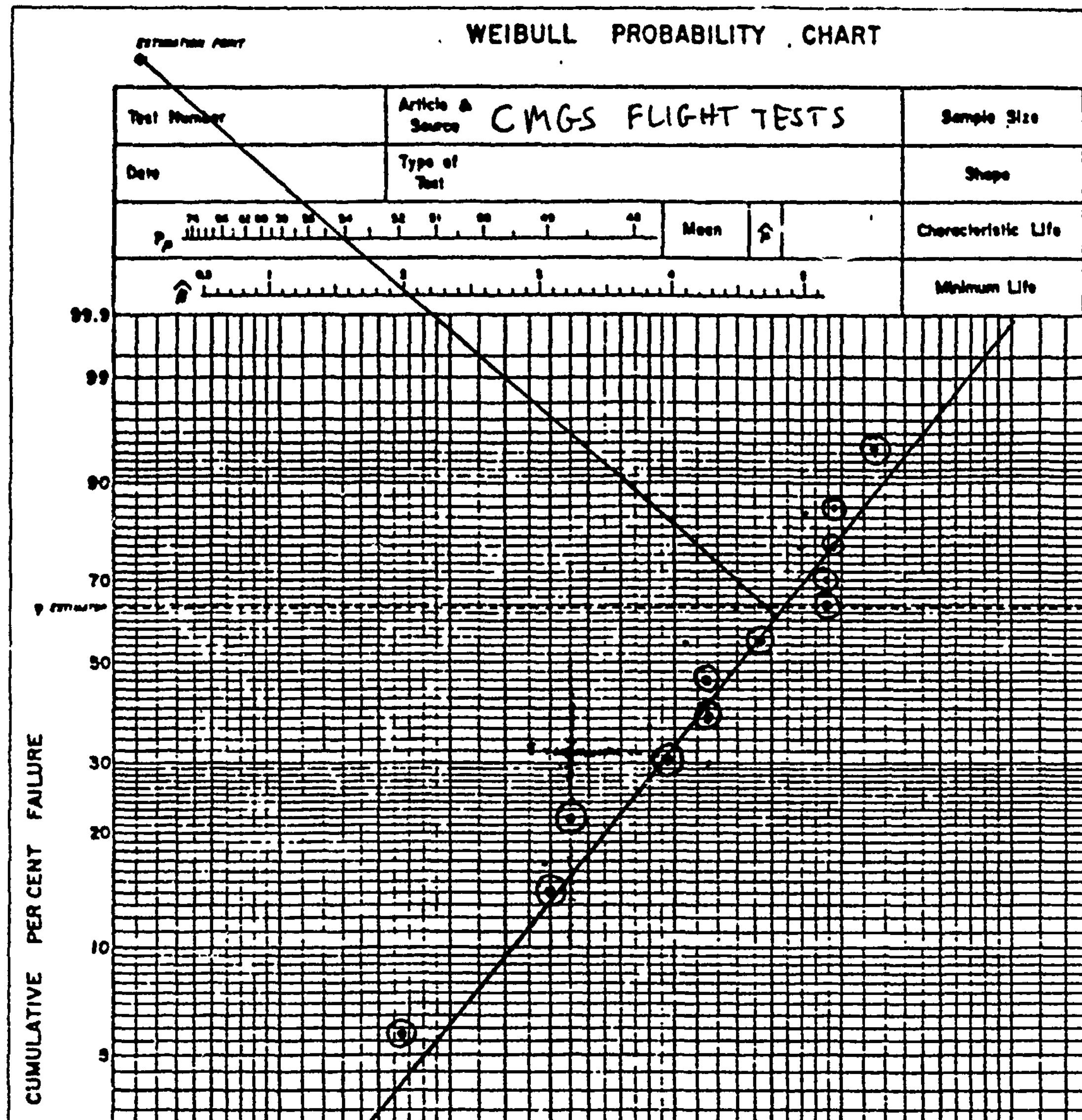


Figure 9. Flight Test Failure Weibull Plot

configurations. However, because the changes were all originally classified as producibility improvements that should not appreciably change the form, fit, or function of the CMGS units, it is difficult to isolate specific reliability improvements to each change. In addition, the contractor had no obligation to include the changes in every unit that was produced.

Based on the differences in failure rates calculated using the different samples of the flight test database, it seems likely that the configuration changes did indeed affect the reliability of the CMGS. For example, the failure rate for the FY84 units is .01830 vice .0680 for the total flight test program using the exponential approach, and .008033 vice .03543 using the binomial approach.

Although every effort was made in designing the CMGS Reliability Demonstration to simulate the operational environments experienced by the CMGS, the differences between the Bayesian and classical approaches indicate some portions of the test environment were more benign than found in the real world. Because the point estimates from the Bayesian approaches are closer to the flight test results than the classical statistical inference approaches, the Bayesian approaches are better tools for evaluating the results of the CMGS Reliability Demonstration.

Other Applications

The final step in this research is the recommendation of further research in the area of Bayesian reliability. Three proposed applications are described in the next sections.

Advanced Cruise Missile. A clear application of this technology is the generalization of the findings to the next generation cruise missiles. The Advanced Cruise Missile (ACM) Navigation and Guidance System has the same approximate degree of complexity as the CMGS and performs an equivalent function. The Bayesian expert information model should be developed for the ACM and used to build prior distributions upon which to base reliability predictions. Because the ACM is projected to have a higher designed reliability, the disadvantages in conducting testing in support of classical statistical inference analyses would be expected to be greater as well.

Avionics Integrity Program (AVIP). One of the chief thrusts of the ASD AVIP program is to change the Air Force's perception of reliability from a reliance on MTBF to an emphasis on the "Failure Free Operational Period". Of course, with a constant failure rate model, such as the exponential distribution, there is always a finite probability of failures. If the hardware can be designed in so as to locate the requ. -ed operating period within an interval of relatively low probability of failure, such as in the tail portion of a failure distribution, the desired result can be obtained.

The use of Bayesian methodology, particularly the expert information approach, can specify shorter confidence intervals for a given amount of test data, resulting in a lower probability of failure for the tails of the distribution. By specifying the required operating period outside of the confidence interval, the very low probabilities of failure can be achieved (see Figure 10).

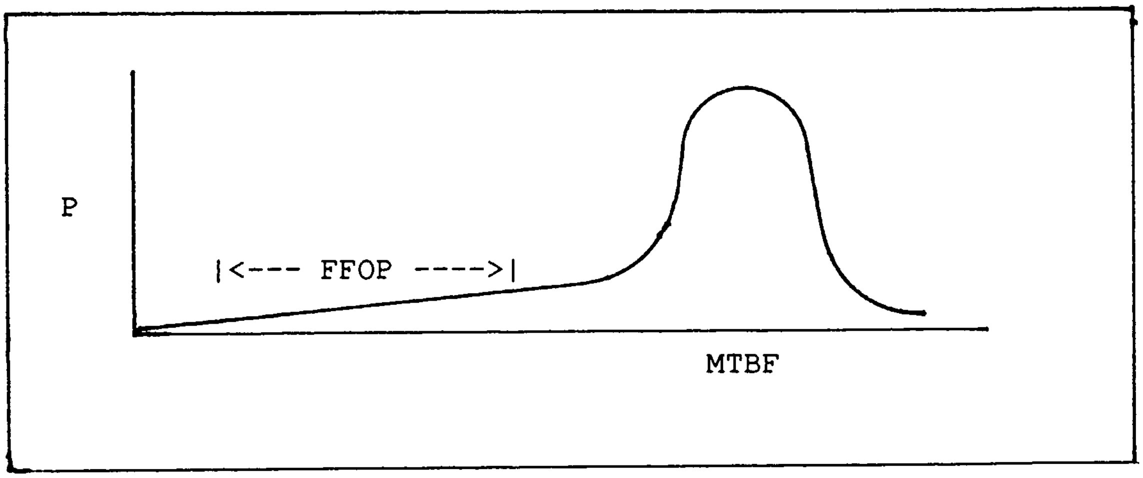


Figure 10. AVIP Failure Free Operational Period (FFOP)

Foreign Technology Evaluation. The Bayesian approaches, in particular the expert information approaches, seem to be good candidates for evaluating the reliability of foreign technology. In many of these scenarios, the analyst has some knowledge of the current threat or capability and considerably more information regarding earlier types or versions. The Bayesian approach allows the analyst to assess the validity of new information and weight the reliability assessment accordingly. As more information is received, the prior distributions can be easily updated, further refining the analysis.

<u>Summary</u>

The Bayesian models defined in this research are capable of specifying shorter confidence intervals about point estimates for cruise missile electronic component failure rates than the traditional classical statistical inference models. The expert information model is particularly useful in that it can easily utilize a wide variety of data sources, enabling more accurate point estimation and confidence interval calculation. Its point estimates were more representative of the flight test results, and thus presumably the true failure rate, than were the classical models. The confidence intervals were clearly shorter than for any of the classical or Bayesian models, making them particularly useful for managerial decision-making. Because there is uncertainty in verifying the constancy of the CMGS flight test failure rate, the expert information binomial model is recommended for future work with the CMGS units.

Appendix A: TLAM Flight Test Data (26)

Date	Missile Flight	Time	MSL Success	CMGS Success	CMGS FY
790717	AL2:1	2.33	S	F	79
790801	AL1:1	2.44	S	S	79
790908	AL3:1	0.02	F	N	79
790929	AL4:1	4.04	S	S	79
791027	AL2:2	3.94	S	S	79
791115	AL6:1	0.02	F	N	79
791206	AL1:2	0.85	F	S	79
791227	AL4:2	0.74	F	S	79
800124	AL7:1	3.90	S	S	79
800208	AL5:1	4.10	S	S	79
800516	T16:1	1.76	S	S	79
800816	T15:1	0.01	F	N	79
801126	T16:2	1.97	S	F	79
810215	T17:3	1.49	S	S	79
810328	T50:1	1.44	S	S	79
810710	T51	0.99	S	S	79
810730	T50:2	0.98	F	F	79
810919	T17:4	1.35	S	S	79
811027	T52	0.99	S	S	79
811107	T54	0.01	F	N	79
811214	T53	0.99	F	S	79
820225	T72:1	1.90	S	F	79
820325	T73:1	1.89	S	F	79
820330	T56	0.01	F	N	79
820519	T74:1	1.96	S	S	79
820521	T55	0.98	F	S	79
820806	T17:5	1.98	S	S	79
820827	T57	1.04	N	S	79
820827	T72:2	1.13	F	F	79
821112	T75:1	1.77	S	S	79
821203	T104	1.07	S	S	79
821217	T74:2	0.01	F	N	79
830223	T72:3	1.92	S	S	79
830306	T58	0.33	F	F	79
830414	T106	1.06	S	S	81
830416	T78:1	1.92	S	S	81
830510	T79	1.30	S	S	79
830603	T102:1	2.13	S	S	79
830607	T75:2	1.98	S	S	81
830727	T97:1	1.81	S	S	82
831003	T78:2	1.33	S	S	82
831015	T81:1	2.05	S	S	82
831027	T84:1	1.75	S	S	79
831114	T102:2	2.10	S	S	82
831119	T98:1	0.30	F	S	82

840123	T103:1	0.42	F	S	82
840213	T207:1	1.10	S	S	82
840224	T208:1	0.66	N	S	82
840301	T72:4	1.70	F	S	82
840403	T293:1	1.93	S	S	82
840404	T16:3	1.42	S	S	82
840405	T294:1	2.07	S	S	82
840413	T208:2	1.75	S	S	82
840619	T175	1.42	S	S	82
840725	T143	1.00	S	S	82
840821	T151	0.01	F	N	82
840918	T303:1	0.01	F	N	82
841110	T185	1.07	S	S	82
841214	T363:1	2.10	S	S	83
850111	T137	1.27	S	S	82
850212	T269:1	1.97	S	S	84
850223	T181	0.88	S	S	82
850309	T176	0.97	S	S	82
850313	T99:1	2.01	S	S	83
850315	T371:1	2.01	S	S	84
850323	T84:2	0.00	N	N	84
850329	T153	0.56	S	S	82
850418	T207:2	1.95	S	S	83
850511	T262:1	0.00	N	N	84
850604	T205:1	2.18	S	S	84
850604	T200:1	2.18	S	S	82
850730	T208:3	1.57	S	S	84
851005	T101:1	1.13	S	S	85
851022	T196:1	1.43	N	F	82
851108	T183:1	0.90	S	S	82
851122	T262:1	1.06	S	S	84
851126	T366:1	1.96	S	F	83
851208	T265:1	0.20	F	N	82
851212	T354:1	0.52	N	S	84
851212	T276:1	0.60	F	F	81
860109	T180:1	1.31	S	S	84
860215	T258:1	2.00	S	S	84
860401	T179:1	0.97	S	S	82
860614	T186:1	0.90	S	S	82
860626	T338:1	2.15	S	S	83
860626	T368:1	2.15	S	S	83
860630	T334:1	0.00	F	N	84
860630	T207:3	1.33	S	S	84
860801	T188:1	1.50	S	S	82
860802	T311:1	1.15	F	F	83
860916	T373:1	0.00	F	N	83
861121	T182:1	1.75	S	S	82
870119	T430:1	1.40	S	S	84
870119	T272:1	1.50	S	S	85
870120	T442:1	1.50	S	S	84

870126	T233	0.65	F	F	82
870401	T314:1	1.23	F	S	83
870427	T594:1	2.03	S	S	85
870429	T627:1	2.03	S	S	84
870430	T78:3	1.28	S	S	86
870710	T830	0.70	S	S	85
870717	T812:1	1.36	S	S	85
870718	T479:1	1.58	S	S	84
870719	T696:1	1.50	S	S	83
870728	T926:1	1.97	S	S	85
870728	T857:1	1.97	S	S	85
870730	T921:1	1.97	S	S	85
870825	T187	1.27	S	S	82
870826	T831	0.80	S	S	85
870924	T101:2	1.22	S	S	82
871009	T186:2	0.00	N	N	82
871103	T186:2	1.15	S	S	82
871114	T673:1	0.00	S	S	85
871211	T178	1.70	S	S	84
871213	T434:1	0.00	S	S	84
880120	T264:1	2.06	S	S	83
880327	T669:1	1.55	S	S	85
880328	T687:1	1.54	S	S	85
880409	T149:1	1.08	S	S	87
880424	T1173:1	1.55	S	S	87
880503	T1203:1	0.81	S	S	86
880504	T1202:1	1.00	S	S	86
880521	T716	0.00			
880524	T1178:1	1.13	S	S	86
052788	T1179:1	1.28	S	S	86
080688	T1572:1	0.08	S	S	86
080988	T429:1	0.37	F	S	84
081088	T254:1	1.79	S	S	83
082388	T980:1	1.72	S	S	86
090388	T1012	1.43	S	S	86
102188	T426	0.50	S	S	84
102788	T681:1	1.50	S	S	86
110888	T669:2	1.94	S	S	85
011089	T174	0.50	S	S	83

Appendix B: ALCM INE Reliability Improvement Warranty Excerpt (34:6)

DOC. NO. 460372
05/06/87

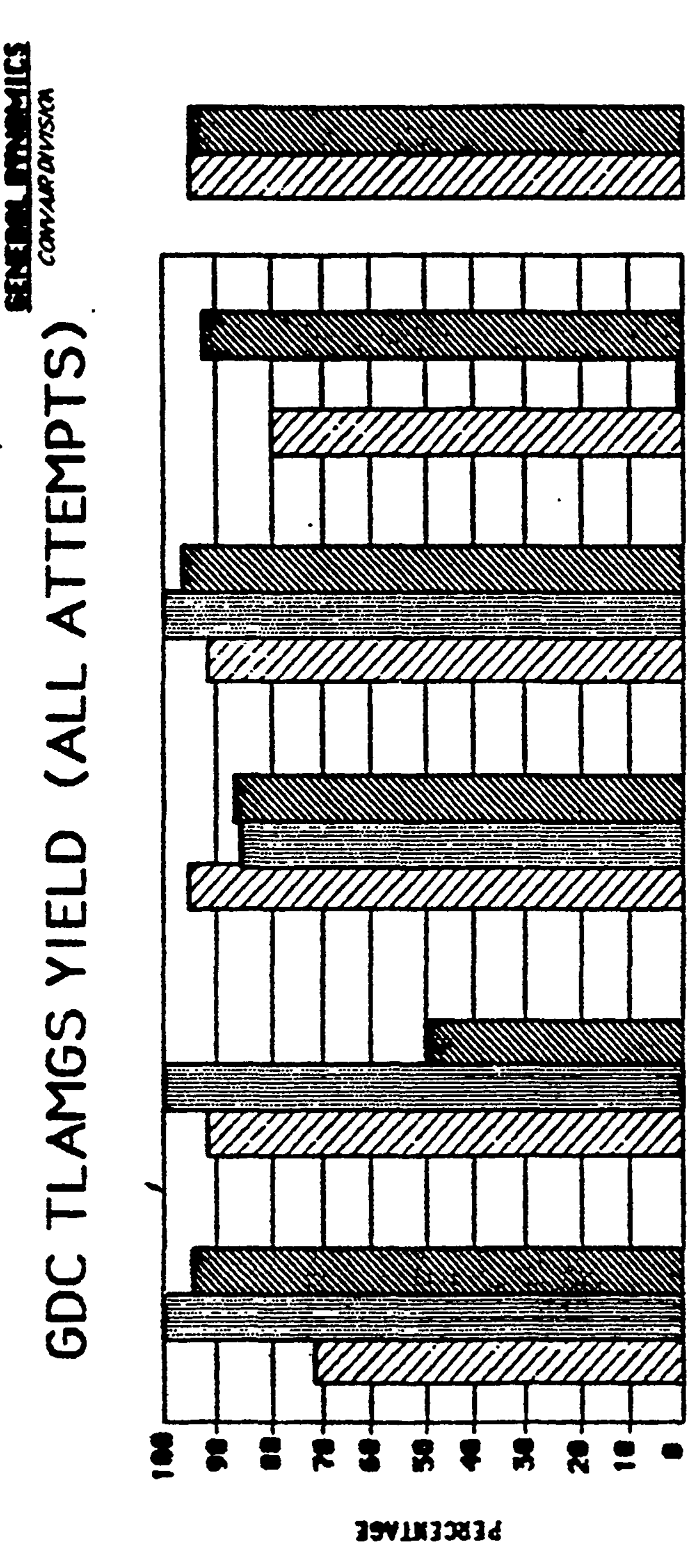
LITTON GUIDANCE & CONTROL SYSTEMS

ALCM INE RELIABILITY IMPROVEMENT WARRANTY (RIW) - UNIT SUMMARY

PROGRAM DETAIL

RIW START DATE	- 30 NOVEMBER 1981	
RIW PERIOD COVERED THIS REPORT	- PERIOD FIVE	
START DATE THIS PERIOD	- 1 JANUARY 1986	
REPORT CUT-OFF DATE	- 31 DECEMBER 1986	
UTILIZATION DETAIL		
AVERAGE QUANTITY OF UNITS INSTALLED	-1663.38	
TOTAL OPERATING HOURS (TOH)-ALL SYSTEMS	- 91,114 HOURS	
AVERAGE OPERATING TIME PER UNIT PER DAY	- 0.150 HOURS	
PERFORMANCE STATISTICS	**ACTUAL**	**GUARANTEED**
ACHIEVED MEAN-TIME-BETWEEN FAILURES	- 424 HOURS	395 HOURS
AVAILABILITY	- 97.0 %	96.0 % EFF.
AVERAGE TURN-AROUND-TIME (TAT)	- 18.0 DAYS	22.0 DAYS
PROGRAM STATUS		
TOTAL WARRANTY DAYS	- 1,858 DAYS	
WARRANTY DAYS USED THRU PERIOD	- 1,858 DAYS	
WARRANTY DAYS REMAINING	- 0 DAYS	

Appendix C: Tomahawk Burn-In Test Report Excerpts
(32:2-8 - 2-9)

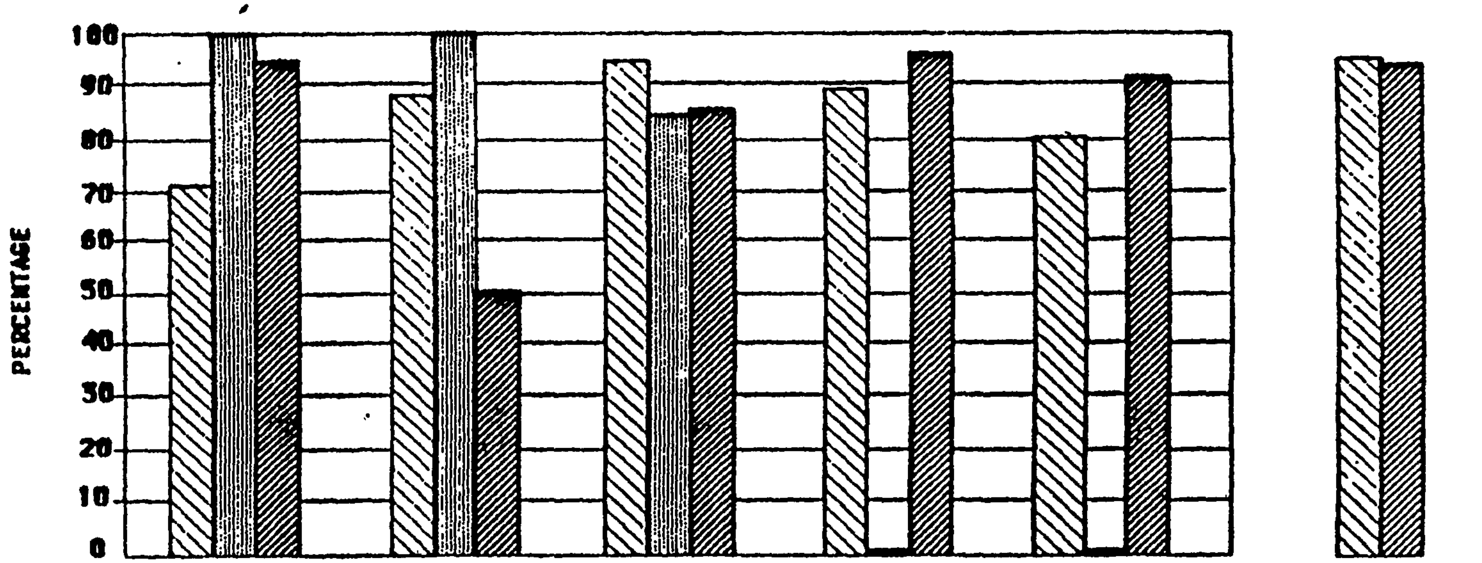


	SEP - NOV 85			DEC 85 - FEB 86			MAR - MAY 86			JUN - AUG 86			SEP - NOV 86			DEC 86 - JAN 87	
	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	MSL LEVEL
PASS	5	3	17	19	8	1	34	17	33	19	2	25	40	0	34	20	20
FAIL	2	0	1	2	0	1	*2	3	5	2	0	1	10	0	3	1	1
YIELD	71%	100%	94%	91%	100%	50%	*94%	85%	87%	91%	100%	96%	80%	0	92%	95%	95%

* DOES NOT INCLUDE THE 3 PRVT FAILURES. YIELD = 86.8% WITH PRVT FAILURES

GENERAL DYNAMICS
CONVAIR DIVISION

GDC TLAMGS YIELD (1ST PASS)

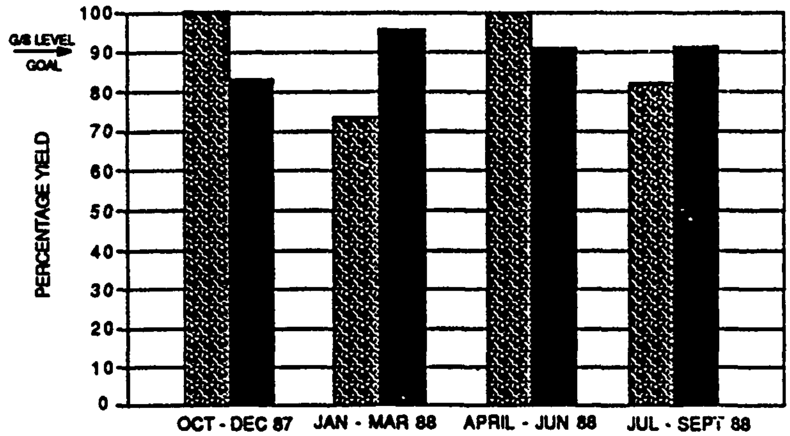


	SEP - NOV 85			DEC 85 - FEB 86			MAR - MAY 86			JUN - AUG 86			SEP - NOV 86			DEC 86 - JAN 87	
	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	PRVT	MSL LEVEL	GS LEVEL	MSL LEVEL
PASS	5	3	16	15	8	1	32	16	29	16	0	24	35	0	31	18	16
FAIL	2	0	1	2	0	1	* 2	3	5	2	0	1	9	0	3	1	1
YIELD	71%	100%	94%	88%	100%	50%	94%	84%	85%	89%	0	96%	80%	0	91%	95%	94%

* DOES NOT INCLUDE THE 3 PRVT FAILURES YIELD = 86.8% WITH PRVT FAILURES

TLAMGS YIELDS
LEVEL 1 VS. LEVEL 2

DESIRED TREND



TLAMS COMPLETED	45	34	11	25	23	21	39	23
FAIL	0	6	3	1	0	2	7	2
YIELD %	100%	82%	73%	96%	100%	90%	82%	91%
LEVEL:								

GS LEVEL (LEVEL 2)

MISSILE LEVEL (LEVEL 1)

FIGURE 2-6

Figure 2-6 above shows TLAM Guidance Set yields for Level 1 and Level 2 testing on a quarterly basis and for this reporting period (July - September 1988) for comparison. This information can be used to discern two things: first, whether yield at Level 2 testing is improving and second, whether Level1 (missile level) yield is less or greater than Level 2 and if that gap is changing.

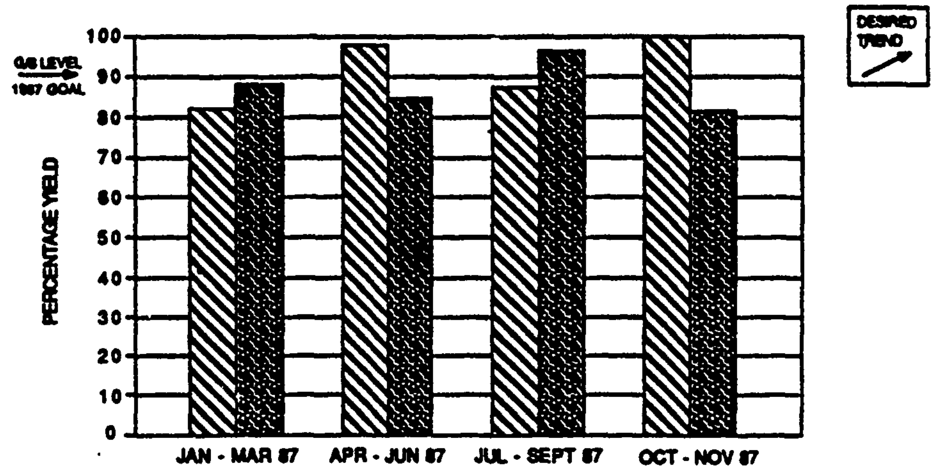
Ground rules for data in this figure are as follows:

- Only first attempt at passing Level 2 ATP and first Level 1 Guidance Set failures considered.
- Only units completing testing during each period considered.
- Only proven or pending hardware failures included.

Level 2 yield is down for the 3rd quarter. The major discrepancy is against RMUCs with four failures. These items have been tested on Litton's TATE and are being dispositioned for return to the vendor. Level 1 yield remains essentially the same this quarter.

Appendix E: Tomahawk Burn-In Test Report Excerpts (34: 2-10)

TLAMGS YIELDS
LEVEL 1 VS. LEVEL 2



TLAMS COMPLETED	27	33	46	40	24	24	35	26
FAIL	5	2	1	6	3	1	0	5
YIELD %	82%	88%	98%	85%	88%	96%	100%	81%
LEVEL:	GS	MSL	GS	MSL	GS	MSL	GS	MSL

FIGURE 2-6

Figure 2-6 above shows TLAM Guidance Set yields for Level 1 and Level 2 testing on a quarterly basis and for this reporting period (October - November) for comparison. This information can be used to discern two things: first, whether yield at Level 2 testing is improving and second, whether Level1 (missile level) yield is less or greater than Level 2 and if that gap is changing.

Ground rules for data in this figure are as follows:

- Only first attempt at passing Level 2 ATP and first Level 1 Guidance Set failures considered.
- Only units completing testing during each period considered.
- Only proven or pending hardware failures included.

The figure shows Level 1 yield decreasing while Level 2 yield is improving to 100%. This disparity between guidance set level testing and missile level guidance removals is not desirable and indicates that Level 2 test escapes (latent defects) are likely being passed along to missile level. Accordingly, the positive trend at Level 2 must be tampered with this concern. Further Level 1 discussion is found back in section 1 under figure 1-5, while Level 2 detail is on the following pages.

Appendix F: Calculations

Flight Test Classical Exponential

All Data

$$\text{Lower Limit} = \frac{15.38}{2 * 176.44} = .04358$$

$$\text{Upper Limit} = \frac{36.42}{2 * 176.44} = .10321$$

FY84

$$\text{Lower Limit} = \frac{.7107}{2 * 54.65} = .006502$$

$$\text{Upper Limit} = \frac{5.991}{2 * 54.65} = .05481$$

Flight Test Classical Binomial

All Data

$$\text{Lower Limit} = \frac{(107+1)*F_{.95}(2*107+2,\ 2*119-2*107)}{119-107+(107+1)*F_{.95}(2*107+2,\ 2*119-2*107)}$$

$$= \frac{108*F_{.95}(216,24)}{12+108*F_{.95}(216,24)} = \frac{108*1.555}{12+108*1.555} = .9333$$

and converting to a failure rate, $R(t) = \exp(-\lambda*3)$,

$$= \ln(.9333)/-3 = .02301$$

$$\text{Upper Limit} = \frac{107}{107+(119-107+1)*F_{.95}(2*119-2*107+2,\ 2*107)}$$

$$= \frac{107}{107+13*F_{.95}(26,214)} = \frac{107}{107+13*1.405} = .8542$$

$$= \ln(R)/(-3) = \ln(.8542)/-3 = .05253$$

FY84

$$\text{Lower Limit} = \frac{(41+1)*F_{.95}(2*41+2,\ 2*42-2*41)}{42-41+(41+1)*F_{.95}(2*41+2,\ 2*42-2*41)}$$

$$= \frac{42*F_{.95}(84,2)}{1+42*F_{.95}(84,2)} = \frac{42*10.1}{1+42*10.1} = .9976$$

$$= \ln(.9976)/-3 = .0007849$$

$$\text{Upper Limit} = \frac{41}{41+(42-41+1)*F_{.95}(2*42-2*41+2,\ 2*41)}$$

$$= \frac{41}{41+2*F_{.95}(4,82)} = \frac{41}{41+2*2.01} = .9107$$

$$= \ln(R)/(-3) = \ln(.9107)/-3 = .03118$$

Classical Exponential

Phase II

$$\text{Lower Limit} = \frac{.1026}{2 * 423.11} = .0001212$$

$$\text{Upper Limit} = \frac{9.488}{2 * 881.413} = .01121$$

Phase I Extension

$$\text{Lower Limit} = \frac{2.733}{2 * 881.413} = .001550$$

$$\text{Upper Limit} = \frac{12.592}{2 * 881.413} = .007142$$

Combined Phases

$$\text{Lower Limit} = \frac{7.962}{2 * 2251} = .001769$$

$$\text{Upper Limit} = \frac{23.68}{2 * 2251} = .005260$$

Classical Binomial

Pnase II

$$\text{Lower Limit} = \frac{(125+1)*F_{.95}(2*125+2,\ 2*126-2*125)}{126-125+(125+1)*F_{.95}(2*125+2,\ 2*126-2*125)}$$

$$= \frac{126*F_{.95}(252,\ 2)}{1+126*F_{.95}(252,\ 2)} = \frac{126*10.2}{1+126*10.2} = .9992$$

and converting to a failure rate, $R(t) = \exp(-\lambda*3)$,

$$= \ln(.9992)/-3 = .0002593$$

$$\text{Upper Limit} = \frac{125}{125+(126-125+1)*F_{.95}(2*126-2*125+2,\ 2*125)}$$

$$= \frac{125}{(125+2*F_{.95}(4,250)} = \frac{125}{125+2*1.96} = .9696$$

$$= \ln(R)/(-3) = \ln(.9696)/-3 = .01029$$

Phase I Extension

$$\text{Lower Limit} = \frac{(261+1)*F_{.95}(2*261+2,\ 2*264-2*261)}{264-261+(261+1)*F_{.95}(2*261+2,\ 2*264-2*261)}$$

$$= \frac{264*F_{.95}(524,\ 6)}{1+126*F_{.95}(524,\ 6)} = \frac{264*2.72}{1+264*2.72} = .9958$$

$$= \ln(.9958)/-3 = .00140$$

$$\text{Upper Limit} = \frac{261}{261+(264-261+1)*F_{.95}(2*264-2*261+2,\ 2*261)}$$

$$= \frac{261}{261+4*F_{.95}(8,522)} = \frac{261}{261+4*1.68} = .9749$$

$$= \ln(R)/(-3) = \ln(.9749)/-3 = .008474$$

<u>Combined</u> <u>Phases</u>

$$\text{Lower Limit} = \frac{(665+1)*F_{.95}(2*665+2,\ 2*672-2*665)}{672-665+(665+1)*F_{.95}(2*665+2,\ 2*672-2*665)}$$

$$= \frac{666*F_{.95}(1332,14)}{7+666*F_{.95}(1332,14)} = \frac{666*1.80}{7+666*1.80} = .9942$$

$$= \ln(.9942)/-3 = .001941$$

$$\text{Upper Limit} = \frac{665}{665+(672-665+1)*F_{.95}(2*672-2*665+2,\ 2*665)}$$

$$= \frac{665}{(665+8*F_{.95}(14,1330)} = \frac{665}{665+8*1.51} = .9822$$

$$= \ln(R)/(-3) = \ln(.9822)/-3 = .006001$$

Bayesian Exponential

Phase II

$$\frac{s+\alpha}{t+\beta} = \frac{1+2}{423.11+75.2} = .006020$$

$$\text{Lower Limit} = \frac{X^2_{.95}(2*1+2*2)}{2*(423.11+75.2)} = \frac{1.635}{996.62} = .001641$$

$$\text{Upper Limit} = \frac{X^2_{.05}(2*1+2*2)}{2*(423.11+75.2)} = \frac{12.59}{996.62} = .01263$$

Phase I Extension

$$\frac{s+\alpha}{t+\beta} = \frac{3+2}{861.413+75.2} = .005338$$

$$\text{Lower Limit} = \frac{X^2_{.95}(2*3+2*2)}{2*(861.413+75.2)} = \frac{3.945}{1873.23} = .002106$$

$$\text{Upper Limit} = \frac{X^2_{.05}(2*3+2*2)}{2*(861.413+75.2)} = \frac{18.31}{1873.23} = .009775$$

Combined Phases

$$\frac{s+\alpha}{t+\beta} = \frac{7+2}{2251+75.2} = .003869$$

$$\text{Lower Limit} = \frac{X^2_{.95}(2*7+2*2)}{2*(2251+75.2)} = \frac{9.39}{4652.4} = .002018$$

$$\text{Upper Limit} = \frac{X^2_{.05}(2*7+2*2)}{2*(2251+75.2)} = \frac{28.87}{4652.4} = .006205$$

Bayesian Binomial

Phase II

$$\frac{125 + 48}{126 + 50} = .9830 \text{ , convert to } \lambda,$$

$$\lambda = \ln(.9830)/-3 = .005731$$

$$\text{Lower Limit} = \frac{(125+48)*F.95(2*125+2*48, 2*126+2*50-2*125-2*48)}{126+50-125-48+(125+48)*F.95(2*125+2*48, 2*126+2*50-2*125-2*48)}$$

$$= \frac{173*F.95(346,6)}{3+173*F.95(346,6)} = \frac{173*2.725}{3+173*2.72} = .9937$$

to convert to λ, $\ln(.9937)/-3 = .002115$

$$\text{Upper Limit} = \frac{125 + 48}{125+48+(126+50-125-48)*F.95(2*126+2*50-2*125-2*48, 2*125+2*48)}$$

$$= \frac{173}{173+3*F.95(6,346)} = \frac{173}{173+3*1.785} = .9700$$

to convert to λ, $\ln(.9700)/-3 = .01016$

Phase I Extension

$$\frac{261 + 48}{264 + 50} = .9841 \text{ , convert to } \lambda,$$

$$\lambda = \ln(.9841)/-3 = .005351$$

$$\text{Lower Limit} = \frac{(261+48)*F.95(2*261+2*48,2*264+2*50-2*261-2*48)}{264+50-261-48+(261+48)*F.95(2*261+2*48,\ 2*264+2*50-2*261-2*48)}$$

$$= \frac{309*F.95(618,10)}{5+309*F.95(618,10)} = \frac{309*2.06}{5+309*2.06} = .9922$$

to convert to λ, ln(.9922)/-3 = .002608

$$\text{Upper Limit} = \frac{261 + 48}{261+48+(264+50-261-48)*F.95(2*264+2*50-2*261-2*48,\ 2*261+2*48)}$$

$$= \frac{309}{309+5*F.95(10,618)} = \frac{309}{309+5*1.618} = .9745$$

to convert to λ, ln(.9745)/-3 = .008615

Combined Phases

$$\frac{665 + 48}{672 + 50} = .9875 \text{ , convert to } \lambda,$$

$$\lambda = \ln(.9874)/-3 = .004181$$

$$\text{Lower Limit} = \frac{(665+48)*F.95(2*665+2*48,2*672+2*50-2*665-2*48)}{672+50-665-48+(665+48)*F.95(2*665+2*48,\ 2*672+2*50-2*665-2*48)}$$

$$= \frac{713*F.95(1426,18)}{9+713*F.95(1426,18)} = \frac{713*1.66}{9+713*1.66} = .9925$$

to convert to λ, ln(.9925)/-3 = .002525

$$\text{Upper Limit} = \frac{665 + 48}{665+48+(672+50-665-48)*F_{.95}(2*672+2*50-2*665-2*48,\ 2*665+2*48)}$$

$$= \frac{713}{713+9*F_{.95}(18,1426)} = \frac{713}{713+9*1.448} = .9821$$

to convert to λ, ln(.9821)/-3 = .006038

Bayesian Expert Information Exponential TBPI Calculations

			Phase I			
$\Sigma(p(\beta\|\gamma i)$	0.00018	0.018777	0.241852	0.545598	0.168585	0.025009
			Phase II			
$\Sigma(p(\beta\|\gamma i)$	0.000576	0.025328	0.244706	0.52407	0.17491	0.030411
			Phase I Extension			
$\Sigma(p(\beta\|\gamma i)$	0.000149	0.016531	0.227239	0.547102	0.180416	0.028564
			Combined Phases			
$\Sigma(p(\beta\|\gamma i)$	3.65E-05	0.016529	0.292403	0.56561	0.115761	0.009661

γi	Ph 1	x=26	Ph 1-ext	x=26	Ph 2	x=27	Combined	x=23
0.0005	0.00018	0.180253	0.000149	0.175747	0.000576	0.221668	3.65E-05	0.133526
0.0006	0.00018	0.21242	0.000149	0.198456	0.000576	0.246081	3.65E-05	0.162763
0.0007	0.00018	0.236587	0.000149	0.221165	0.000576	0.270494	3.65E-05	0.192
0.0008	0.00018	0.260755	0.000149	0.243874	0.000576	0.322844	3.65E-05	0.221236
0.0009	0.00018	0.315296	0.000149	0.29857	0.000576	0.375193	3.65E-05	0.250473
0.001	0.00018	0.369838	0.000149	0.353265	0.000576	0.427542	3.65E-05	0.27971
0.0011	0.00018	0.42438	0.000149	0.40796	0.000576	0.479892	3.65E-05	0.308946
0.0012	0.00018	0.478922	0.000149	0.462655	0.000576	0.532241	3.65E-05	0.365504
0.0013	0.00018	0.533463	0.000149	0.517351	0.000576	0.584591	3.65E-05	0.422061
0.0014	0.00018	0.588005	0.000149	0.572046	0.000576	0.63694	3.65E-05	0.478619
0.0015	0.018777	0.642547	0.016531	0.626741	0.025328	0.68929	0.016529	0.535176
0.0016	0.018777	0.695229	0.016531	0.679798	0.025328	0.739164	0.016529	0.590084
0.0017	0.018777	0.747911	0.016531	0.732855	0.025328	0.789038	0.016529	0.644992
0.0018	0.018777	0.800593	0.016531	0.785913	0.025328	0.803996	0.016529	0.6999
0.0019	0.018777	0.815574	0.016531	0.802301	0.025328	0.818954	0.016529	0.754809
0.002	0.018777	0.830555	0.016531	0.81869	0.025328	0.833912	0.016529	0.809717
0.0021	0.018777	0.845535	0.016531	0.835078	0.025328	0.848871	0.016529	0.864625
0.0022	0.018777	0.860516	0.016531	0.851467	0.025328	0.863829	0.016529	0.874548
0.0023	0.018777	0.875497	0.016531	0.867855	0.025328	0.878787	0.016529	0.884471
0.0024	0.018777	0.890478	0.016531	0.884244	0.025328	0.893745	0.016529	0.894394
0.0025	0.241852	0.905459	0.227239	0.900632	0.244706	0.908703	0.292403	0.904318
0.0026	0.241852	0.898132	0.227239	0.89535	0.244706	0.901724	0.292403	0.886653
0.0027	0.241852	0.890805	0.227239	0.891267	0.244706	0.894744	0.292403	0.868989
0.0028	0.241852	0.883479	0.227239	0.886585	0.244706	0.873315	0.292403	0.851325
0.0029	0.241852	0.861794	0.227239	0.866718	0.244706	0.851885	0.292403	0.833661
0.003	0.241852	0.84011	0.227239	0.84685	0.244706	0.830456	0.292403	0.815996
0.0031	0.241852	0.818426	0.227239	0.826982	0.244706	0.809026	0.292403	0.798332
0.0032	0.241852	0.796742	0.227239	0.807115	0.244706	0.787597	0.292403	0.770058
0.0033	0.241852	0.775057	0.227239	0.787247	0.244706	0.766167	0.292403	0.741784
0.0034	0.241852	0.753373	0.227239	0.76738	0.244706	0.744738	0.292403	0.713509
0.0035	0.545598	0.731689	0.547102	0.747512	0.52407	0.723309	0.56561	0.685235
0.0036	0.545598	0.67963	0.547102	0.695658	0.52407	0.673943	0.56561	0.62964
0.0037	0.545598	0.627571	0.547102	0.643805	0.52407	0.624577	0.56561	0.574045
0.0038	0.545598	0.575512	0.547102	0.591951	0.52407	0.57217	0.56561	0.51845
0.0039	0.545598	0.520952	0.547102	0.537241	0.52407	0.519763	0.56561	0.462855
0.004	0.545598	0.466393	0.547102	0.48253	0.52407	0.467356	0.56561	0.40726
0.0041	0.545598	0.411833	0.547102	0.42782	0.52407	0.414949	0.56561	0.351666
0.0042	0.545598	0.357273	0.547102	0.37311	0.52407	0.362542	0.56561	0.295105
0.0043	0.545598	0.302713	0.547102	0.3184	0.52407	0.310135	0.56561	0.238543

0.0044	0.545598	0.248154	0.547102	0.26369	0.52407	0.257728	0.56561	0.181982
0.0045	0.168585	0.193594	0.180416	0.20898	0.17491	0.205321	0.115761	0.125421
0.0046	0.168585	0.176735	0.180416	0.190938	0.17491	0.18783	0.115761	0.113845
0.0047	0.168585	0.159877	0.180416	0.172896	0.17491	0.170339	0.115761	0.102269
0.0048	0.168585	0.143018	0.180416	0.154855	0.17491	0.152848	0.115761	0.090693
0.0049	0.168585	0.12616	0.180416	0.136813	0.17491	0.135357	0.115761	0.079117
0.005	0.168585	0.109301	0.180416	0.118772	0.17491	0.117866	0.115761	0.067541
0.0051	0.168585	0.092443	0.180416	0.10073	0.17491	0.100375	0.115761	0.055965
0.0052	0.168585	0.075584	0.180416	0.082688	0.17491	0.082884	0.115761	0.044389
0.0053	0.168585	0.058726	0.180416	0.064647	0.17491	0.065393	0.115761	0.032813
0.0054	0.168585	0.041867	0.180416	0.046605	0.17491	0.047902	0.115761	0.021237
0.0055	0.025009	0.025009	0.028564	0.028564	0.030411	0.030411	0.009661	0.009661
0.0056	0.025009	0.022508	0.028564	0.025707	0.030411	0.02737	0.009661	0.008695
0.0057	0.025009	0.020007	0.028564	0.022851	0.030411	0.024329	0.009661	0.007729
0.0058	0.025009	0.017506	0.028564	0.019995	0.030411	0.021288	0.009661	0.006763
0.0059	0.025009	0.015005	0.028564	0.017138	0.030411	0.018247	0.009661	0.005797
0.006	0.025009	0.012504	0.028564	0.014282	0.030411	0.015205	0.009661	0.00483
0.0061	0.025009	0.010004	0.028564	0.011425	0.030411	0.012164	0.009661	0.003864
0.0062	0.025009	0.007503	0.028564	0.008569	0.030411	0.009123	0.009661	0.002898
0.0063	0.025009	0.005002	0.028564	0.005713	0.030411	0.006082	0.009661	0.001932
0.0064	0.025009	0.002501	0.028564	0.002856	0.030411	0.003041	0.009661	0.000966

Bayesian Expert Information Binomial TBPI Calculations

Phase I

$\Sigma(p(\beta\|\gamma i)$	2.03E-05	0.00423	0.109058	0.492242	0.304215	0.090234

Phase II

$\Sigma(p(\beta\|\gamma i)$	0.000214	0.013039	0.174326	0.516565	0.238501	0.057355

Phase I Extension

$\Sigma(p(\beta\|\gamma i)$	1.91E-05	0.004053	0.106385	0.488931	0.307683	0.092929

Combined Phases

$\Sigma(p(\beta\|\gamma i)$	2.01E-07	0.000456	0.040409	0.391317	0.40066	0.167158

γi	Ph 1	x=29	Ph 1-ext	x=29	Ph 2	x=28	Combined	x=26
0.0005	2.03E-05	0.113308	1.91E-05	0.110457	0.000214	0.170146	2.01E-07	0.028742
0.0006	2.03E-05	0.16253	1.91E-05	0.159348	0.000214	0.187558	2.01E-07	0.032783
0.0007	2.03E-05	0.211753	1.91E-05	0.208239	0.000214	0.239193	2.01E-07	0.036824
0.0008	2.03E-05	0.260975	1.91E-05	0.257131	0.000214	0.290828	2.01E-07	0.040865
0.0009	2.03E-05	0.310197	1.91E-05	0.306022	0.000214	0.342463	2.01E-07	0.079997
0.001	2.03E-05	0.359419	1.91E-05	0.354913	0.000214	0.394098	2.01E-07	0.119128
0.0011	2.03E-05	0.408641	1.91E-05	0.403804	0.000214	0.445733	2.01E-07	0.15826
0.0012	2.03E-05	0.457864	1.91E-05	0.452695	0.000214	0.497368	2.01E-07	0.197392
0.0013	2.03E-05	0.507086	1.91E-05	0.501587	0.000214	0.549003	2.01E-07	0.236524
0.0014	2.03E-05	0.556308	1.91E-05	0.550478	0.000214	0.600638	2.01E-07	0.275655
0.0015	0.00423	0.60553	0.004053	0.599369	0.013039	0.652273	0.000456	0.314787
0.0016	0.00423	0.635529	0.004053	0.629732	0.013039	0.702626	0.000456	0.353873
0.0017	0.00423	0.665527	0.004053	0.660095	0.013039	0.725172	0.000456	0.392959
0.0018	0.00423	0.695526	0.004053	0.690458	0.013039	0.747718	0.000456	0.432045
0.0019	0.00423	0.725524	0.004053	0.720821	0.013039	0.770265	0.000456	0.472066
0.002	0.00423	0.755523	0.004053	0.751184	0.013039	0.792811	0.000456	0.512086
0.0021	0.00423	0.785521	0.004053	0.781547	0.013039	0.815357	0.000456	0.552107
0.0022	0.00423	0.81552	0.004053	0.81191	0.013039	0.837904	0.000456	0.592127
0.0023	0.00423	0.845518	0.004053	0.842273	0.013039	0.86045	0.000456	0.632147
0.0024	0.00423	0.875517	0.004053	0.872636	0.013039	0.882996	0.000456	0.672168
0.0025	0.109058	0.905515	0.106385	0.902999	0.174326	0.905542	0.040409	0.712188
0.0026	0.109058	0.903633	0.106385	0.901653	0.174326	0.91196	0.040409	0.748213
0.0027	0.109058	0.90175	0.106385	0.900307	0.174326	0.900263	0.040409	0.784238
0.0028	0.109058	0.899868	0.106385	0.898962	0.174326	0.888566	0.040409	0.820264
0.0029	0.109058	0.897986	0.106385	0.897616	0.174326	0.876868	0.040409	0.832938
0.003	0.109058	0.896103	0.106385	0.896271	0.174326	0.865171	0.040409	0.845613
0.0031	0.109058	0.894221	0.106385	0.894925	0.174326	0.853474	0.040409	0.858288
0.0032	0.109058	0.892339	0.106385	0.89358	0.174326	0.841777	0.040409	0.870963
0.0033	0.109058	0.890456	0.106385	0.892234	0.174326	0.83008	0.040409	0.883638
0.0034	0.109058	0.888574	0.106385	0.890889	0.174326	0.818383	0.040409	0.896313
0.0035	0.492242	0.886692	0.488931	0.889543	0.516565	0.806686	0.391317	0.908988
0.0036	0.492242	0.837468	0.488931	0.84065	0.516565	0.760765	0.391317	0.886572
0.0037	0.492242	0.788243	0.488931	0.791757	0.516565	0.709108	0.391317	0.864156
0.0038	0.492242	0.739019	0.488931	0.742864	0.516565	0.657452	0.391317	0.84174
0.0039	0.492242	0.689795	0.488931	0.693971	0.516565	0.605795	0.391317	0.802608
0.004	0.492242	0.640571	0.488931	0.645078	0.516565	0.554139	0.391317	0.763476
0.0041	0.492242	0.591346	0.488931	0.596184	0.516565	0.502482	0.391317	0.724345
0.0042	0.492242	0.542122	0.488931	0.547291	0.516565	0.450826	0.391317	0.685213
0.0043	0.492242	0.492898	0.488931	0.498398	0.516565	0.399169	0.391317	0.646081
0.0044	0.492242	0.443674	0.488931	0.449505	0.516565	0.347513	0.391317	0.606949
0.0045	0.304215	0.39445	0.307683	0.400612	0.238501	0.295856	0.40066	0.567818

0.0046	0.304215	0.364028	0.307683	0.369844	0.238501	0.272006	0.40066	0.527752
0.0047	0.304215	0.333607	0.307683	0.339075	0.238501	0.248156	0.40066	0.487686
0.0048	0.304215	0.303185	0.307683	0.308307	0.238501	0.224306	0.40066	0.44762
0.0049	0.304215	0.272763	0.307683	0.277539	0.238501	0.200456	0.40066	0.407554
0.005	0.304215	0.242342	0.307683	0.246771	0.238501	0.176605	0.40066	0.367488
0.0051	0.304215	0.21192	0.307683	0.216002	0.238501	0.152755	0.40066	0.327422
0.0052	0.304215	0.181499	0.307683	0.185234	0.238501	0.128905	0.40066	0.287356
0.0053	0.304215	0.151077	0.307683	0.154466	0.238501	0.105055	0.40066	0.24729
0.0054	0.304215	0.120656	0.307683	0.123698	0.238501	0.081205	0.40066	0.207224
0.0055	0.090234	0.090234	0.092929	0.092929	0.057355	0.057355	0.167158	0.167158
0.0056	0.090234	0.081211	0.092929	0.083636	0.057355	0.051619	0.167158	0.150442
0.0057	0.090234	0.072187	0.092929	0.074344	0.057355	0.045884	0.167158	0.133726
0.0058	0.090234	0.063164	0.092929	0.065051	0.057355	0.040148	0.167158	0.11701
0.0059	0.090234	0.054141	0.092929	0.055758	0.057355	0.034413	0.167158	0.100295
0.006	0.090234	0.045117	0.092929	0.046465	0.057355	0.028677	0.167158	0.083579
0.0061	0.090234	0.036094	0.092929	0.037172	0.057355	0.022942	0.167158	0.066863
0.0062	0.090234	0.02707	0.092929	0.027879	0.057355	0.017206	0.167158	0.050147
0.0063	0.090234	0.018047	0.092929	0.018586	0.057355	0.011471	0.167158	0.033432
0.0064	0.090234	0.009023	0.092929	0.009293	0.057355	0.005735	0.167158	0.016716

Bibliography

1. Arsenault, J.E. and J.A. Roberts. Reliability and Maintainability of Electronic Systems. Rockville MD: Computer Science Press, Inc., 1980.

2. Barlow, R. E. Combining Component and System Information in System Reliability Calculation. Technical Report. AFOSR-TR-86-0169. University of California, Berkeley CA, 1986 (AD-A167090).

3. Berger, James O. Statistical Decision Theory. New York: Springer-Verlag New York Inc., 1980.

4. Box, George E. P. and George C. Tiao. Bayesian Inference in Statistical Analysis. Reading MA: Addison-Wesley Publishing Company, Inc., 1973.

5. Chay, S.C. "Use of Bayesian Method in Reliability Estimation," 1984 IEEE Proceedings Annual Reliability and Maintainability Symposium, 16: 216-221 (1984).

6. Coppola, Anthony. "Bayesian" Reliability Tests Made Practical. Technical Report. RADC-TR-81-106. Rome Air Development Center (RBET), Griffis AFB NY, July 1981 (AD-A108150).

7. Department of the Air Force. USAF R & M 2000 Process. Washington DC: HQ USAF, 5 October 1987.

8. Department of Defense. Military Standard: Reliability Program for Systems and Equipment Development and Production. MIL-STD-785B. Washington DC: Government Printing Office, 15 September 1980.

9. GLCM Weapon System Assessment Program Final Evaluation Report. Technical Report. GWSAP-26406. Contract F33657-87-C-0044, performed by McDonnell Douglas Corporation for Aeronautical Systems Division, Wright-Patterson AFB OH, 30 September 1987.

10. Gnedenko, B. V. and others. Mathematical Methods of Reliability Theory. New York: Academic Press, 1969.

11. GWSAP - FCII Flight Critical Item Test and Inspection Assessment. Technical Report. GWSAP-26578 Rev A. Contract F33657-87-C-0044, performed by McDonnell Douglas Corporation for Aeronautical Systems Division, Wright-Patterson AFB OH, 4 December 1987.

12. Johnson, Wesley and Jessica Utts. "Bayesian Robust Estimation of the Mean," Applied Statistics, 35: 8-17 (1986).

13. Kachigan, Sam Kash. Statistical Analysis. New York: Radius Press, 1986.

14. Kaplan, Stan. The Bayesian Approach to Data Reduction in Probabilistic Risk Analysis, September 1981. PLG-0205. Pickard, Lowe and Garrick, Inc., Newport Beach CA.

15. -----. A Methodology for Assessing the Reliability of Boxes. Technical Report. PLG-0651. Contract F33657-85-C-0265, performed by Pickard, Lowe and Garrick, Inc. for Aeronautical Systems Division, Wright-Patterson AFB OH, August 1988.

16. -----. "On a 'Two-Stage' Bayesian Procedure for Determining Failure Rates From Experiential Data," IEEE Transactions on Power Apparatus and Systems, PAS-102: 1-11 (January 1983).

17. -----. Quantitative Reliability Assessment Using the Bayesian Approach. Technical Report. PLG-0622. Contract DAA D05-87-0083, performed by Pickard, Lowe and Garrick, Inc. for U.S. Department of the Army, Fort Rucker AL, April 1988.

18. Lampkin, Harold and Alan Winterbottom. "Approximate Bayesian Intervals for the Reliability of Series Systems from Mixed Subsystem Test Data," Naval Research Logistics Quarterly, 30: 313-317 (1983).

19. Launer, Robert L. and Nozer D. Singpurwalla. "A Combined Bayes-Sampling Theory Method for Monitoring a Bernoulli Process," (AD-P003840).

20. Lewis, E. E. Introduction to Reliability Engineering. New York: John Wiley & Sons, Inc., 1987.

21. Lynn, I. Hugh, Computer Resources Branch Chief. Personal interviews. ASD/ENASC, Wright-Patterson AFB OH, 15 May - 7 July 1989.

22. Martz, Harry F. and Ray A. Waller. Bayesian Reliability Analysis. New York: John Wiley & Sons, Inc., 1982.

23. McCrory, Jim. "Helicopter Reliability Assessment," Army Research, Development & Acquisition Magazine, 20: 19-20 (1985).

24. Morgan, Bruce W. An Introduction to Bayesian Statistical Decision Processes. Englewood Cliffs NJ: Prentice-Hall, Inc., 1968.

25. Nelson, Wayne. Applied Life Data Analysis. New York: John Wiley & Sons, Inc., 1982.

26. Shaver, Danny, Advanced Cruise Missile Reliability Program Manager. Personal interview. Aeronautical Systems Division, Wright-Patterson AFB OH, 5 January 1989.

27. Sinha, S.K. and B.K. Kale. Life Testing and Reliability Estimation. New Dehli: Wiley Eastern Limited, 1980.

28. Tomahawk BGM-109G Ground Launch Cruise Missile Guidance Set Reliability and Field Effect Upon Reliability Test Phase I Extension Report. Technical Report. TMHK-25983-3. Contract F33657-87-C-044, performed by McDonnell Douglas Astronautics Company for Aeronautical Systems Division, Wright-Patterson AFB OH, 25 September 1987.

29. Tomahawk BGM-109G Ground Launch Cruise Missile Guidance Set Reliability and Field Effect Upon Reliability Test Phase I Report. Technical Report. TMHK-25983-1. Contract F33657-87-C-044, performed by McDonnell Douglas Astronautics Company for Aeronautical Systems Division, Wright-Patterson AFB OH, 24 August 1987.

30. Tomahawk BGM-109G Ground Launch Cruise Missile Guidance Set Reliability and Field Effect Upon Reliability Test Phase II Report. Technical Report. TMHK-25983-2. Contract F33657-87-C-044, performed by McDonnell Douglas Astronautics Company for Aeronautical Systems Division, Wright-Patterson AFB OH, 10 September 1987.

31. Tomahawk BGM-109G Ground Launch Cruise Missile Guidance Set Reliability and Field Effect Upon Reliability Test Plan. Technical Report. TMHK-24412. Contract F33657-87-C-044, performed by McDonnell Douglas Astronautics Company for Aeronautical Systems Division, Wright-Patterson AFB OH, 15 November 1986.

32. Tomahawk Cruise Missile Quarterly Burn-In Test Report. Technical Report. GDC-SLCM-86-058. Contract N00032-84-C-4484, performed by General Dynamics/Convair Division for Cruise Missile Project, Washington DC, February 1987.

33. _Tomahawk Cruise Missile Quarterly Burn-In Test Report_. Technical Report. GDC-SLCM-86-058. Contract N00032-84-C-4484, performed by General Dynamics/Convair Division for Cruise Missile Project, Washington DC, December 1987.

34. _Tomahawk Cruise Missile Quarterly Burn-In Test Report_. Technical Report. GDC-SLCM-86-058. Contract N00032-87-C-3102, performed by General Dynamics/Convair Division for Cruise Missile Project, Washington DC, October 1988.

35. _Warranty Data Report for the Inertial Navigation Element FY1983 & 1984 INE RIW/AG_. Technical Report. 450372, Rev I. Contracts N00019-81-3113 and N00032-84-C-4234, performed by Litton Guidance & Control Systems for Joint Cruise Missile Project, Washington, D.C., 8 May 1987.

36. Winterbottom, Alan. _Approximating Posterior Distributions of System Reliability Derived From Component Test Data_. Technical Report. DAJA-82-C-0736. European Research Office of the U.S. Army, London England, October 1984 (AD-A152057).

37. Wonnacott, Thomas H. "Bayesian and Classical Hypothesis Testing," _Journal of Applied Statistics_. _13_: 149-157 (1986).

38. Wood, Maj Buddy B. _Bayesian Reliability Test Plans for One-Shot Devices_. Technical Report. USAFA-TR-84-01. Department of Mathematical Sciences, U.S. Air Force Academy CO, September 1983 (AD-A138234).

Vita

Captain Richard K. Lemaster was born on 3 October 1960 in Boulder, Colorado. He graduated from high school in Freehold, New Jersey, in 1978 and attended Duke University in Durham, North Carolina while on an Air Force ROTC scholarship. He graduated with the degree of Bachelor of Science in Chemistry in 1982. His first assignment was with the Air Force Element of the Joint Cruise Missiles Project in Washington D.C. The Navy was the executive service until the Ground Launched Cruise Missile (GLCM) program split away and transitioned to Aeronautical Systems Division (ASD), Wright Patterson AFB OH. While at ASD, his positions included the GLCM Reliability and Quality Improvement Program Manager, where he implemented the GLCM Weapon System Assessment Program, and the Advanced Cruise Missile Systems Analysis Program Manager. He matriculated to the Air Force Institute of Technology School of Systems and Logistics in May 1988. He currently resides in Bellbrook OH with his wife Barbara and new daughter Katie.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFIT/GSM/LSG/89S-24

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
School of Systems and Logistics

6b. OFFICE SYMBOL (If applicable)
AFIT/LSY

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS (City, State, and ZIP Code)
Air Force Institute of Technology
Wright-Patterson AFB OH 45433-6583

7b. ADDRESS (City, State, and ZIP Code)

8a. NAME OF FUNDING/SPONSORING ORGANIZATION
Advanced Cruise Missile SPO

8b. OFFICE SYMBOL (If applicable)
ASD/VCA

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State, and ZIP Code)
ASD/VCA
Wright-Patterson AFB OH 45433

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
APPLICATION OF BAYESIAN RELIABILITY CONCEPTS TO CRUISE MISSILE ELECTRONIC COMPONENTS

12. PERSONAL AUTHOR(S)
Richard K. Lemaster, B.S., Captain USAF

13a. TYPE OF REPORT
MS Thesis

13b. TIME COVERED
FROM ________ TO ______

14. DATE OF REPORT (Year, Month, Day)
1989 September

15. PAGE COUNT
124

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
12	03	
16	04	01

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Bayes Theorem, Probability, Statistical Inference, Cruise Missiles

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Thesis Advisor: Phillip E. Miller, Lt Col, USAF
Assistant Professor
Department of Logistics Management

Approved for public release: IAW AFR 190-1.

Larry W. Emmelhainz

LARRY W. EMMELHAINZ, Lt Col, USAF 14 Oct 89
Director of Research and Consultation
Air Force Institute of Technology (AU)
Wright-Patterson AFB OH 45433-6583

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Phillip E. Miller, Lt Col, USAF

22b. TELEPHONE (Include Area Code)
(513)255-4149

22c. OFFICE SYMBOL
LSG

The purpose of this ~~research~~ thesis was to evaluate the applicability of Bayesian statistical methods to the problem of determining cruise missile component reliability. There were three objectives:

1) to develop models incorporating Bayesian reliability concepts that can be used to predict component reliability based on data available in a program transitioning from development to production;

2) to determine the model's validity in comparison with classical statistical models; and

3) to assess the accuracy of both approaches against actual cruise missile flight test history.

A total of six models were developed for the failure rate of the Tomahawk Cruise Missile Guidance Set using both exponential and binomial distributions. The flight test data seemed to belong to another failure distribution, and was not useful as a measure of performance as had been proposed.

The Bayesian Expert Information Model provided reasonable point estimates of the failure rate and markedly shorter 90% confidence intervals. In general, the Bayesian models had confidence intervals that were shorter than the classical statistical inference models, allowing a more accurate decision-making process.

Future effort in this field could be directed toward applying these models to other weapon systems or components. Other applications could include using the Bayesian approach in the Aeronautical Systems Division Avionics Integrity Program (AVIP) to increase the Failure Free Operating Period of components, or in the assessment of the reliability of foreign technology based on partial information.